Reg. No. : | | | | | | | | | | |

Question Paper Code: 12061

M.E. DEGREE EXAMINATION, OCTOBER 2014.

First Semester

Structural Engineering

01PMA125 - APPLIED MATHEMATICS

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Find the Laplace transform of cos at.

2. Find the Fourier sine transform of sin at.

3. Write down the Laplace equation in spherical polar coordinates.

4. Write down any two properties of a harmonic function.

5. Write down the Euler's equation for the extremum of the functional given by

$$I\left[y(x)\right] = \int_0^1 \left(xy + y^2 - 2y^2 y'\right) dx, \qquad y(0) = 1,\ y(1) = 2.$$

6. State the necessary condition for a functional to attain its extremum.

7. If the characteristic polynomial $P(\lambda)$ of a matrix A is such that $P(0) = 0$, then what is the value of $|A|$. Justify your answer.

8. Define a skew-Hermitian matrix and give an example.

9. Define Hermite polynomial.

10. Write down two-point Gaussian quadrature formula.

PART - B (5 x 16 = 80 Marks)

11. (a) Using Laplace transform, solve the one dimensional wave equation

$u_{xx} = u_{tt}, 0 < x < 1, t > 0$

subject to the conditions $u(0, t) = u(1, t) = 0$ *for* $t > 0$ and $u(x, 0) = \sin \pi x$,

$u_t(x, 0) = -\sin \pi x$ *for* $0 < x < 1$. (16)

Or

(b) Use Fourier transform to solve the heat conduction equation given by

$$k\,\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, t > 0.$$

subject to the boundary conditions $u(x, t)$ and $u_t(x, t)$ both $\to 0$ *as* $|x| \to 0$ and the initial condition $u(x, 0) = f(x), -\infty < x < \infty$. (16)

12. (a) Using the Fourier transform method, show that the solution of the two-dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

is valid in the half-plane y > 0, subject to the condition

$$u(x,0) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$$

and $\lim(x^2 + y^2) \to 0$ in the above half plane. (16)

Or

(b) Solve the Poisson equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2$ subject to the boundary conditions given by u(0, y) = 0, u(5, y) = 0, u(x, 0) = 0 and u(x, 4) = 0. (16)

13. (a) Find the extremum of the function $I\left[y(x)\right] = \int_{x_0}^{x_1} \frac{\left(1 + y'^2\right)^{1/2}}{x}\, dx.$

(16)

Or

(b) (i) Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$. (8)

(ii) Find the extremum of the functional $I[y(x)] = \int_0^1 (y'^2 + y^2)\,dx,\ y(0) = 0,\ y(1) = 1.$ using the Rayleigh-Ritz method. (8)

14. (a) Using Faddeev-Leverrier method, find the resolvent of the matrix

$$A = \begin{bmatrix} -2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$ (16)

Or

(b) Find all the Eigen values of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

by using Power method. (16)

15. (a) (i) Use Gaussian three-point formula and evaluate $I = \int_1^5 \frac{dz}{z}$. (8)

(ii) Evaluate $\int_0^{\frac{\pi}{2}} \sin t\,dt$, by Gaussian two-point formula. (8)

Or

(b) (i) Evaluate $\int_1^2 \int_1^2 \frac{1}{x+y}\,dxdy$, by Gaussian quadrature formula. (8)

(ii) Approximate the integral $\int_0^{\frac{\pi}{4}} x^2 \sin x\,dx$, using Gaussian quadrature with n = 2 and compare your results with the exact value of the integral. (8)