Reg. No. : | | | | | | | | | | |

Question Paper Code: 12021

M.E. DEGREE EXAMINATION, OCTOBER 2014.

First Semester

Communication Systems

01PMA122 - APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Regulation 2013)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Prove that $J_n(x) - \frac{n}{x} J_n(x) = -J_{n+1}(x)$.

2. Using the generating function prove that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.

3. Determine whether the following matrix is positive definite or not.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$

4. Define Toeplitz matrix.

5. The number of hardware failures of a computer system in a week of operations has the following p m f:

Number of failures	0	1	2	3	4	5	6
Probability	0.18	0.28	0.25	0.18	0.06	0.04	0.01

Find the mean of the number of failures in a week.

6. Define Moment Generating function.

7. Let (x, y) be a two dimensional random variable. Define covariance of (x, y). if **x** and y are independent , what will be the covariance of (x,y) ?

8. If the joint p d f of (X,Y) is given by f(x,y) = 2-x-y,in $0 \le x < y \le 1$, find E(X).

9. What are the basic characteristics of a queuing system?

10. Consider an M/ M/ 1 queuing system, if $\lambda = 6$ and $\mu = 8$, find the probability of at least 10 customers in the system.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Show that $J_{5/2}(x) = \sqrt{\frac{2}{n\pi}}\left(\frac{3-x^2}{x^2}\ \ sinx - \frac{3}{x}\ \ cosx\right)$. (8)

(ii) Prove that $\frac{d}{dx}(x^{-n}J_n(x)) = -x^{-n}J_{n+1}(x)$. (8)

Or

(b) (i) $\int x\, J_0{}^2\ dx = \frac{1}{2}x^2\left[J_0{}^2(x) + J_1^2(x)\right] + c$. Where c is a constant. (8)

(ii) Prove that $\int_0^\infty\ \ e^{-ax}J_o(bx)dx = \frac{1}{\sqrt{a^2+b^2}}$. . (8)

12. (a) Determine the Cholesky decomposition for

$$A = \begin{bmatrix} 16 & -3 & 5 & -8 \\ -3 & 16 & -5 & -8 \\ 5 & -5 & 24 & 0 \\ -8 & -8 & 0 & 21 \end{bmatrix}$$ (16)

Or

(b) Using Q.R factorization method compute all the eigen values of the matrix

$$A = \begin{bmatrix} 6 & 4 & 1 & 1 \\ 4 & 6 & 1 & 1 \\ 1 & 1 & 5 & 2 \\ 1 & 1 & 2 & 5 \end{bmatrix}$$ (16)

13. (a) (i) Define Normal Distribution. Find MGF, mean and variance. (8)

(ii) A Machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are (a) exactly 3 defectives, (b) not more than 3 defectives. (8)

Or

(b) (i) The daily consumption of milk in a city in excess of 10000 gallons is approximately exponential distributed with parameter $\theta = 1000$. The city has a daily stock of 20000 gallons. What is the probability that the stock is insufficient for both days if two days are selected at random? (8)

(ii) A random variable 'X' has a uniform distribution over (-3,3). Compute

(a) $P(X < 2)$, $P(|X| < 2)$, $P(|X - 2| < 2)$,

(b) Find K for which $P(X > K) = \frac{1}{3}$. (8)

14. (a) (i) If X and Y have the joint p.d.f $f(x, y) = \begin{cases} \frac{3}{4} + xy, & 0 < x < 1, 0 < y < 1 \\ 0, & other\ wise \end{cases}$

Find $f(y/x)$ and $P(y>1/2/X = 1/2)$ (8)

(ii) Find Karl Pearson's correlation co- efficient between X and Y from the following data:

X :	78	89	97	69	59	79	61	61
Y :	125	137	156	112	107	136	123	108

(8)

Or

(b) Calculate the co - efficient of correlation between x and y from the following table and write down the regression equation of y on x

Y \ X	0 – 40	40 – 80	80 – 120	120 – 160
10 – 30	9	4	1	-
30 – 50	47	19	6	-
50 – 70	26	18	11	-
70 – 90	2	3	2	2

(16)

15. (a) (i) A Supermarket has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour.

Find

(a) The probability that the cashier is idle.

(b) The average number of customers in the queueing systems.

(c) The average time a customer spends in system.

(d) The average number of customers in the queue.

(e) The average time a customer spends in the queue waiting for service. (8)

(ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(a) Find the effective arrival rate at the clinic.

(b) What is the expected waiting time until a patient is discharged from the clinic. (8)

Or

(b) (i) A barber shop has two barbers and three chairs for customers. Assume that customers arrive in a poisson fashion at the rate of 5 per hour, and that each barber services customers according to an exponential distribution with mean of 15 minutes. Further, If a customer arrives and there are no empty chairs in the shop, he will leave. What is the probability that the shop is empty? (8)

(ii) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time (the time taken to hump a train) distribution is also exponential with an average of 36 minutes Calculate

(a) Expected queue size (line length)

(b) Probability that the queue size exceeds 10.

If the input of trains increases to an average of 33 per day, what will be the change in (a) and (b). (8)