Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 45878

5 Years M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Third Semester

Software Engineering

XCS 231/10677 SW 301 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 year M.Sc. Computer Technology and M.Sc. Information Technology)

(Regulation 2003/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from the relation $z^2 = (x - a^2) + (y - b^2)$.

2. Distinguish between complete solution and general solution of a partial differential equation.

3. State the formula to get the sum of the Fourier series of a function $f(x)$ in $(c, c + 2l)$ at the point α where α is an interior point discontinuity of $f(x)$.

4. State Parseval's theorem for the function $y = f(x)$ defined in $(-\pi, \pi)$.

5. Find the Fourier transform of $e^{-a|x|}\cos bx$ using the result Fourier cosine transform of e^{-ax} is $\frac{a}{s^2 + a^2}$, $a > 0$.

6. State Fourier Integral theorem.

7. Evaluate $\int_o^t t \sin 3t \, dt$ using Laplace Transform.

8. Given that $L(t \sin t) = \frac{2s}{(s^2 + 1)^2}$ find $L(t \sin at)$.

9. State final value theorem on Z-transform.

10. Compute $Z\left(\frac{1}{n!}\right)$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Form the differential equation by eliminating the arbitrary functions f and g from $z = f(x+iy)+(x+iy)g(x-iy)$, where $i=\sqrt{-1}$ and $x+iy \neq z$. (8)

(ii) Solve $4xyz = pq + 2px^2y + 2qxy^2$. (8)

Or

(b) (i) Solve $(y+z)p+(z+x)q = x+y$. (8)

(ii) Solve $\left(9D^{2^2} + 6DD' + (D')^2\right)Z = \left(e^x + e^{-2y}\right)^2$. (8)

12. (a) (i) Find the Fourier series expansion of $f(x) = x(1-x)(2-x)$ in (0, 2). Deduce the sum of the series $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} \ldots\ldots \infty$. (10)

(ii) Find the half-range sine series of $f(x) = \sin ax$ in $(0, l)$. (6)

Or

(b) (i) Find the half-range cosine series of $f(x) = x(\pi - x)$ in $(0, \pi)$. Hence, find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots\ldots \alpha$. (10)

(ii) Verify Dirichlet's conditions for the function $f(x) = x^2$ defined in $(0, 2l)$. (6)

13. (a) (i) Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 1-|x| & , for |x| < 1 \\ 0 & , for\ |x| > 1 \end{cases}$

Hence prove that $\int_0^{\alpha} \left(\frac{\sin x}{x}\right)^4 dx = \frac{\pi}{3}$. (8)

(ii) Solve for $f(x)$, given that $\int_0^{\alpha} f(x) \sin xt\ dt = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$ (8)

Or

(b) (i) Find the Fourier transform cosine transform of x^{n-1}. (8)

(ii) Prove that the Fourier transform of the convolution of two functions is the product of their Fourier transforms. (8)

14. (a) (i) Find the Laplace transform of $\dfrac{e^{at} - \cos bt}{t}$. (8)

(ii) Using convolution theorem, evaluate the inverse Laplace transform of $\dfrac{s}{\left(s^2 + a^2\right)^2}$. (8)

Or

(b) (i) Find the inverse Laplace transform of $\tan^{-1}\left(\dfrac{2}{s^2}\right)$. (8)

(ii) Find the Laplace transform of the "full-sine wave rectifier" function $f(t)$, defined as $f(t) = |\sin \omega t|$, $t \geq 0$, $f(t)$ is periodic with period $\dfrac{\pi}{\omega}$. (8)

15. (a) (i) Compute $Z(\sin n\theta)$ and hence find $Z\left(e^{-an} \sin n\theta\right)$. (8)

(ii) Using convolution theorem, find the inverse Z-transform of $\left(\dfrac{z}{z-a}\right)^3$. (8)

Or

(b) (i) Find the inverse Z – transform of $\dfrac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. (8)

(ii) Find $Z(u_{n+2})$ if $Z(u_n) = \dfrac{z}{z-1} + \dfrac{z}{z^2+1}$. (8)

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