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Question Paper Code : 45252

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Third Semester

Software Engineering

EMA 003 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL
TRANSFORMS

(Regulations 2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the constants from $z = ax + by + \sqrt{a^2 - b^2}$.
2. Find the order and degree of the partial differential equation.
$$\frac{\partial^4 z}{\partial x^4} + 5 \frac{\partial^3 z}{\partial^2 x \partial y} + 3 \left(\frac{\partial^2 z}{\partial y^2} \right)^2 = 0.$$
3. If $f(x)$ is an even function in $(-\pi, \pi)$, give the Fourier series and the formula for Fourier coefficient.
4. State Parseval's theorem for Fourier series.
5. Find the Fourier Cosine transform of e^{-x} .
6. Prove that $F_s [x f(x)] = -\frac{d}{ds} F_c(s)$.
7. State the shifting properties of Laplace transformation.
8. Evaluate $L[e^{-2t} \sin 5t]$.
9. State the convolution property of Z transform.
10. Find $Z[(n+1)(n+2)]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation from
 $f(2x + 3y + 4z, x^2 + y^2 + z^2) = 0.$ (8)

(ii) Solve $9(p^2z + q^2) = 4.$ (8)

Or

(b) (i) Solve $(mz - ny)p + (nx - ly)q = ly - mx.$ (8)

(ii) Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y.$ (8)

12. (a) (i) Find the Fourier series expansion of $f(x) = x + x^2$ in $(-\pi, \pi).$ (8)

(ii) Obtain the half range cosine series for $f(x) = x(2 - x)$ in $0 \leq x \leq 2$ and hence evaluate $\frac{1}{1^2} - \frac{1}{2^3} + \frac{1}{3^2} - \frac{1}{4^3} + \dots$ (8)

Or

(b) (i) If $f(x) = \begin{cases} x, & 0 < x < l \\ 2l - x, & l < x < 2l \end{cases}$ expand $f(x)$ as a Fourier series. (8)

(ii) Expand $f(x) = x$ in a half range sine series in the range $0 < x < l$ and hence find $\sum_1^\infty \frac{1}{n^2}.$ (8)

13. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$. Hence, prove that

$$\int_0^\infty \left(\frac{\sin s - s \cos s}{s^3} \right) \cos\left(\frac{s}{2}\right) ds = \left(\frac{3\pi}{16} \right).$$

Or

(b) Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

14. (a) (i) Determine $L^{-1} \left[\log \frac{(s^2 - 1)}{s(s + 1)} \right].$ (8)

(ii) Using Laplace transform evaluate $\int_0^\infty te^{-3t} \cos 5t dt.$ (8)

Or

(b) (i) Find $L\left[\frac{\cos 2t - \cos 5t}{t}\right]$. (8)

(ii) Evaluate $L^{-1}\left[\frac{2}{(s+1)(s^2+4)}\right]$. (8)

15. (a) (i) Evaluate using convolution theorem $Z^{-1}\left[\frac{4z^2}{(z-5)(z-9)}\right]$. (8)

(ii) Find inverse Z transform of $\left[\frac{2z}{(z-1)(z^2+1)}\right]$. (8)

Or

(b) (i) Find the inverse Z - transform of $\frac{z(z+1)}{(z-1)^3}$. (8)

(ii) If $Z[f(t)] = F(z)$ then $Z[f(t+T)] = z[F(z) - f(0)]$. (8)