Reg. No. :

Question Paper Code : 45873

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Second Semester

Software Engineering

XCS 122/10677 SW 202 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to 5 Year M.Sc. Information Technology and M.Sc. Computer Technology)

(Regulation 2003/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^2\int_0^1 4xy\,dx\,dy$.

2. Evaluate $\int_0^1\int_0^2\int_0^3 dx\,dy\,dz$.

3. Show that $\overline{F} = (2x + yz)\overline{i} + (xz - 3)\overline{j} + xy\overline{k}$ is irrotational.

4. If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and S the surface of a sphere of unit radius, find $\iint_S \overline{r} \cdot \overline{ds}$.

5. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and passing through the origin.

6. Find the equation of the sphere whose center is $(4, 4, -2)$ and which passes through the origin.

7. Prove that an analytic function with constant real part is constant.

8. Verify whether $e^x \sin y$ is harmonic.

9. State Cauchy's integral formula.

10. Find the residue of cot z at its poles.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Change the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\,dx$ and then evaluate it. (8)

(ii) Using triple integral, find the volume of the tedrahedran bounded by the co-ordinate planes and the plane $x+y+z=1$. (8)

Or

(b) (i) Using double integral, find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (8)

(ii) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z} dz\,dy\,dx$. (8)

12. (a) (i) Find the directional derivative of $P = 2xy + z^2$ at the point $(1, -1, 3)$ in the direction of $\bar{i} + 2\bar{j} + 2\bar{k}$. (8)

(ii) Find the work done by the force $\overline{F} = 3xy\bar{i} - y^2\bar{j}$, when it moves a particle along the curve $y = 2x^2$ in the $xy-$ plane from (0, 0) to (1, 2). (8)

Or

(b) Verify Green's theorem in plane with respect to $\int_C \left(x^2dx - xydy\right)$, where C is the boundary of the square formed by $x=0, y=0, x=a$ and $y=a$

. (16)

13. (a) (i) Find the length and equation of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-1} = \frac{y+7}{2} = \frac{z-6}{4}$. (8)

(ii) Show that the two spheres $x^2+y^2+z^2-2x-4y-4z=0$ and $x^2+y^2+z^2+10x+2z+10=0$ touch each other. (8)

Or

(b) (i) Prove that the lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-1}{-2}$ and $\frac{x-2}{2} = \frac{y-3}{-1} = \frac{z+4}{3}$ are coplanar and find the equation of the plane containing them. (8)

(ii) Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as a great circle. (8)

14. (a) (i) If $u(x,y)$ and $v(x,y)$ are harmonic in a region R, prove that the function $(u_y - v_x) + i(u_x + v_y)$ is an analytic function of z. (8)

(ii) Find the analytic function $f(z) = u + iv$ given that $u + v = \frac{2x}{x^2+y^2}$ and $f(1) = i$. (8)

Or

(b) (i) If $u = x^2 - y^2$ and $v = -\frac{y}{x^2+y^2}$, prove that both u and v satisfy Laplace equation, but $u + iv$ is not analytic function of z. (8)

(ii) If $f(z)$ is analytic function of z prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) | f(z) |^2 = 4 | f'(z) |^2$. (8)

15. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$ where C is the circle $| z | = 4$. (8)

(ii) Find the Laurant's series expansion of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ in the region $2 < | z | < 3$. (8)

Or

(b) (i) State and prove Cauchy's integral theorem. (8)

(ii) Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$. (8)
