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Reg. No. : [ ]

Question Paper Code : 45251

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Second Semester

Software Engineering

EMA 002 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.

2. Change the order of integration in $\int_0^{a}\int_x^{a} f(x, y)\,dy\,dx$.

3. Show that $\vec{f} = (x + 2y)\vec{i} + (y + 3z)\vec{j} + (x - 2z)\vec{k}$ is solenoidal.

4. State Stoke's theorem.

5. Find the equation of the line passing through (1, –2, –1) and perpendicular to the plane $2x - 3y + 5z + 4 = 0$.

6. Find the centre and radius of the sphere $2(x^2 + y^2 + z^2) + 6x - 6y + 8z + 9 = 0$.

7. Verify whether the function $e^x \sin y$ is harmonic (or) not.

8. Prove that the function $f(z) = \bar{z}$ is not analytic.

9. Expand $f(z) = \cos z$ about $z = -\pi/2$ in Taylor's series.

10. Find the singularities of $f(z) = \dfrac{z^2 + 4}{z^3 + 2z^2 + 2z}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\iint_R \frac{e^{-y}}{y}\, dx\, dy$, by choosing the order of integration suitably, given that R is the region bounded by the lines $x = 0$, $x = y$ and $y = \infty$. (8)

(ii) Evaluate $\iiint_V dx\, dy\, dz$, where V is the finite region of space formed by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + 4z = 12$. (8)

Or

(b) (i) Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dy\, dx$ and then evaluate it. (8)

(ii) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dz\, dy\, dx$. (8)

12. (a) (i) Find the values of the constants a, b, c so that $\vec{f} = (axy + b z^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ may be irrotational. For these values of a, b, c, find also the scalar potential of $\vec{f}$. (8)

(ii) Find the directional derivative of $\phi = 2xy + z^2$ at the point (1, –1. 3) in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. (8)

Or

(b) Verify Green's theorem in a plane with respect to $\int_c (x^2\, dx - xy\, dy)$, where C is the boundary of the square formed by $x = 0$, $y = 0$, $x = a$, $y = a$.

13. (a) Find the equation of the image of the line $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$.

Or

(b) Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. Find also their points of contact.

14. (a) (i) If $w = u(x, y) + iv(x, y)$ is an analytic function, the curves of the family $u(x, y) = a$ and the curves of the family $v(x, y) = b$ cut orthogonally where a and b are varying constants. (8)

(ii) Find the analytic function $w = u + iv$ if $v = e^{2x}(x \cos 2y - y \sin 2y)$. Hence, find u. (8)

Or

(b) If $f(z) = u + iv$ is a regular function of z prove that $\nabla^2 \{\log |f(z)|\} = 0$.

15. (a) (i) Evaluate $\int_c \frac{z\,dz}{(z-1)(z-2)^2}$, where C is the circle $|z - 2| = \frac{1}{2}$, using Cauchy's integral formula. (8)

(ii) Find the Laurent's series of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ valid in the region

(1) $|z| < 2$ and (2) $2 < |z| < 3$. (8)

Or

(b) (i) Use Cauchy's residue theorem to evaluate $\int_c \frac{dz}{\sin hz}$, where C is the circle $|z| = 4$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{5 - 3\cos\theta} d\theta$, using contour integration. (8)