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Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 45250

5 Year M.Sc. DEGREE EXAMINATION, JANUARY 2015.

First Semester

Software Engineering

EMA 001 — TRIGNOMETRY, ALGEBRA AND CALCULUS

(Common to 5 Year M.Sc. Software Systems)

(Regulation 2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $\cosh^2 x - \sinh^2 x = 1$.

2. Find the real and imaginary parts of the complex number $\frac{(2+5i)}{(4-3i)}$.

3. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ 1 & -5 & 8 \end{bmatrix}$.

4. Find the sum and product of the eigenvalues of $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$.

5. Find $\frac{du}{dt}$, if $u = e^{xy}$, where $x = \sqrt{a^2 - t^2}$ and $y = \sin^3 t$.

6. If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial(u, v)}{\partial(r, \theta)}$.

7. Find the value of $\int_0^{\frac{\pi}{2}} \sin^5\theta \cos^4\theta \, d\theta$.

8. Evaluate $\int_0^{\infty} \frac{dx}{(a^2+x^2)^{3/2}}$.

9. Find the complimentary function of $(D^4+3D^2-4)y = e^x + \cos(3x)$.

10. Find the particular integral of $(D^4+3D^2-4)y = e^x + \cos(3x)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Using De Moivre's theorem, express $\cos^4\theta \sin^3\theta$ in a series of sines of multiples θ.

(ii) Separate into real and imaginary parts of $\tan^{-1}(\alpha + i\beta)$.

Or

(b) (i) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i\sin\left(\frac{\pi}{2^r}\right)$, prove that $x_1 x_2 x_3 \ \ldots \infty = -1$.

(ii) If ω is a imaginary cube root of unity, prove that $\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega} = 0$.

12. (a) (i) Find for what values of k, the equations $x+y+z=1$, $x+2y+3z=k$, $x+5y+9z=k^2$ have a solution? For these values of k, find the solution also.

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and hence find A^{-1}.

Or

(b) Reduce the quadratic form $2x^2+2y^2+2z^2-2xy-2yz+2zx$ to canonical form through orthogonal transformation and write its reduced form. (16)

13. (a) (i) If $u = xyz$, $v = xy + yz + zx$ and $w = x + y + z$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

(ii) Find the minimum value of $x^2 + y^2 + z^2$, when $x + y + z = 3a$.

Or

(b) (i) Find the Taylor's series expansion of $e^x \cos(y)$ in the neighbourhood of the point $(1, \pi/4)$ upto the second degree term.

(ii) Given the transformation $u = e^x \cos(y)$ and $v = e^x \sin(y)$ and that f is a function of u and v and also of x and y, prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$.

14. (a) (i) Show that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{1 + 2\cos\theta} = \frac{1}{\sqrt{3}} \log(2 + \sqrt{3})$.

(ii) Evaluate $\int_\alpha^\beta \sqrt{(x - \alpha)(\beta - x)}\, dx$, $(\beta > \alpha)$.

Or

(b) (i) Show that the curve $a^2 y^2 = x^2(a^2 - x^2)$ consist of two loops whose area is $2a^3/3$.

(ii) If the curve $(a - x)y^2 = a^2 x$ revolves about its asymptote, find the volume formed.

15. (a) (i) Solve : $(D^2 - 4)y = x^2 \cosh 2x$.

(ii) Solve : $(x^2 D^2 - 2xD - 4)y = 32(\log x)^2$.

Or

(b) (i) Solve the simultaneous equations : $Dx + y = \sin t$, $x + Dy = \cos t$, given that $x = 2$, $y = 0$ at $t = 0$.

(ii) Solve : $[(x + 1)^2 D^2 + (x + 1)D + 1]y = 4\cos(\log(x + 1))$.

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