**Reg. No. :** | | | | | | | | | | |

## Question Paper Code: 53001

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2017

Third Semester

Civil Engineering

15UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, EIE, Mechanical Engineering, Chemical Engineering, Biomedical Engineering and Agricultural Engineering Branches)

(Regulation 2015)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The sum of the Fourier series for the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $(0, 2\pi)$ at $x = \frac{\pi}{2}$ is

   (a) $\frac{\pi}{2}$ (b) $\pi$ (c) 0 (d) $\frac{\pi}{4}$

2. Find the RMS value of the function $f(x) = x$ in the interval $(0, l)$

   (a) $\frac{l}{2}$ (b) $\frac{l}{3}$ (c) $\frac{l}{\sqrt{3}}$ (d) $\frac{l}{\sqrt{2}}$

3. The Fourier Sine transform of $e^{-3x}$ is

   (a) $\sqrt{\frac{2}{\pi}}\left(\frac{a}{s^2 + a^2}\right)$ (b) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2 + 3^2}\right)$ (c) $\sqrt{\frac{2}{\pi}}\left(\frac{3}{s^2 - 3^2}\right)$ (d) $\sqrt{\frac{2}{\pi}}\left(\frac{3}{s^2 + 3^2}\right)$

4. If $F(x)$ is the Fourier Transform of $f(x)$ then $F[f(x)\cos ax] =$

(a) $\frac{1}{2}[F(s+a)+F(s-a)]$ (b) $\frac{1}{2}[F(s+a)-F(s-a)]$

(c) $\frac{1}{2}[F_s(s+a)+F_s(s-a)]$ (d) $\frac{1}{2}[F_s(s+a)-F_s(s-a)]$

5. Find the value of $Z(n)$

(a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{z}{(z-1)^2}$ (d) $\frac{z}{(z+1)^2}$

6. The difference equation of $y = a + b(3)^n$ is

(a) $y_{n+2} - 4y_{n+1} + 3y_n = 0$ (b) $y_{n+2} + 4y_{n+1} + 3y_n = 0$

(c) $y_{n+2} + 4y_{n+1} - 3y_n = 0$ (d) $y_{n+2} + 3y_{n+1} + 4y_n = 0$

7. The PDE obtained by eliminating the functions of $z = f(x+ct) + g(x-ct)$

(a) $r = c^2 t$ (b) $r + t = 0$ (c) $r - t = 0$ (d) $t = rc^2$

8. The Particular Integral of $(D^2 - 2DD' + 2D'^2)z = \sin(x-y)$

(a) $\frac{-\sin(x-y)}{5}$ (b) $\frac{\sin(x-y)}{5}$ (c) $\frac{\sin(x+y)}{5}$ (d) $5\sin(x+y)$

9. Classify the PDE $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$

(a) Elliptic (b) Parabolic (c) Hyperbolic (d) None of these

10. In the wave equation $u_{tt} = c^2 u_{xx}$, what does $c^2$ stand for?

(a) $\frac{T^2}{m}$ (b) $\frac{T}{m}$ (c) $\frac{T^2}{m^2}$ (d) $\frac{T}{m^2}$

PART - B (5 x 2 = 10 Marks)

11. What are the constants $a_0$ & $a_n$ in the Fourier series expansion of $f(x) = x - x^3, (-\pi, \pi)$.

12. Find the Fourier sine transform of $f(x) = \frac{1}{x}$

13. Find the value of $Z\left(\frac{1}{n(n+1)}\right)$.

14. Form the PDE by eliminating the arbitrary constants 'a' and 'b' from $z=(x+a)^2 + (y-b)^2$.

15. A rod 30 cm long has its ends A and B kept at 20 and 80 degree Celsius respectively. Find the steady state temperature distribution in the rod.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Expand in Fourier series of $f(x) = x \sin x$ for $0 < x < 2\pi$ and deduce the result $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots = \frac{\pi - 2}{4}$ (8)

(ii) Obtain the Half range sine series for $f(x) = x, (0, l)$ (8)

Or

(b) (i) Find the complex form of Fourier Series of $f(x) = e^{-x}, -1 \le x \le 1$ (8)

(ii) Find the first two harmonics of the Fourier Series from the following table:

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Y | 9 | 18 | 24 | 28 | 26 | 20 |

(8)

17. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, if\ |x| < 1 \\ 0\ , \quad if\ |x| > 1 \end{cases}$ and hence find the value of $\int_0^\infty \left(\frac{\sin t}{t}\right) dt$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$ (16)

Or

(b) Find the Fourier Sine Transform and Fourier Cosine transform of $f(x) = e^{-ax}, a > 0$. Hence evaluate $\int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$ and $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b)^2}$. (16)

18. (a) (i) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z-1)}$ by convolution theorem. (8)

(ii) Find the Z Transform of $\{a^n\}$ and $\{na^n\}$. (8)

Or

(b) (i) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n, y_0 = y_1 = 0$ using Z Transforms. (8)

(ii) If $U(z) = \frac{z^3 + z}{(z-1)^3}$, find the value of $u_0, u_1$ and $u_2$. (8)

19. (a) (i) Find the singular integral of $z = px + qy + p^2 - q^2$. (8)

(ii) Solve $(D^3 - 2D^2D')z = e^{x+2y} + 4\sin(x+y)$. (8)

Or

(b) (i) Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ (8)

(ii) Solve $x^2p^2 + y^2q^2 = z^2$. (8)

20. (a) A square plate is bounded by the line $x = 0, x = a, y = 0, y = a$. The square plane bounded by the lines $x = a, y = 0, y = a$ are kept at temperature $0^oC$ and the side $x = 0$ is kept at temperature given by $u(0, y) = 100, 0 < y < a$. Find the temperature distribution of the plate. (16)

Or

(b) A uniform string is stretched and fastened to two points *l* apart. Motion is started by displacing the string into the form of the curve *y= kx (l-x)* and then releasing it from this position at time *t= 0*. Find the displacement of the string at a distance *x* from one end at time *t*. (16)

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