Reg. No. : | | | | | | | | | | |

Question Paper Code: 53002

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2017

Third Semester

Computer Science Engineering

15UMA322 - PROBABILITY, STATISTICS AND QUEUEING SYSTEMS

(Common to Information Technology branch)

(Regulation 2015)

(Statistical tables are may be permitted)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If X is a Random variable then, $\text{var}(aX + b) =$

   (a) $aVar(X) + b$ (b) $a^2Var(X) + b$ (c) $a^2Var(X)$ (d) $aVar(X)$

2. Suppose X is continuous RV with PDF $f(x) = c(4x - 2x^2), 0 < x < 2$, then the value of '*c*' is

   (a) 1/2 (b) 3/5 (c) 2/9 (d) 3/8

3. The Joint distribution function of X and Y is given by $F(x, y) = (1 - e^{-x})(1 - e^{-y}), x > 0, y > 0$. Is X and Y independent?

   (a) Yes (b) NO (c) Both (a) and (b) (d) None of these

4. Suppose x and y are _____, then cov(x, y)= 0

   (a) Identical (b) Distinct (c) Dependent (d) Independent

5. The science of experimental design is associated with the name

   (a) Latin square (b) RBD (c) Latin cubes (d) None of these

6. In a 4 x 4 Latin square, the total of possibilities are

   (a) 200 (b) 576 (c) 29 (d) 10

7. The process in which customer refuses to join the queue as it is too long is called

(a) Balking (b) Reneging (c) Priority (d) Jockeying

8. The effective arrival rate λ' is given by

(a) $\mu(1-\rho)$ (b) $\mu(1-P_0)$ (c) $\mu(1+\rho)$ (d) $\mu(1+P_0)$

9. An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars/hr and may wait in the facilities parking lot if the bay is busy. Find Ls if the service time is constant and equal to 10 minutes.

(a) 1/3 (b) 2/3 (c) 1 (d) 4/3

10. In the model $(M/G/1)$ if the service time follows exponential distribution then the model reduces to

(a) Model I (b) Model II (c) Model III (d) Model IV

PART - B (5 x 2 = 10 Marks)

11. For a triangular wave distribution $f(x) = \begin{cases} x, 0 < x < 1 \\ 2-x, 1 < x < 2 \end{cases}$, find the mean?

12. State any one form of central limit theorem.

13. What are the uses of analysis of variance?

14. In (M/M/1) : (k/FCFS) model λ= 3 /hr and μ = 4 / hr and $P_0 = 0.28$. What is the effective arrival rate of a customer?

15. Define a two stage tandem queue.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Three urns contain 6 Green, 4 Black and 4 Green 6 Black and 5 green and 5 black balls. An urn is selected at random and one ball is drawn and is found to be Green. Find the probability that it is drawn from the second urn? (8)

(ii) Find the moment generating function of an Poisson distribution and hence find the mean and variance. (8)

Or

(b) (i) The amount of time that a watch will run without having to be reset is a RV having exponential distribution with mean 120 days. Find the probability that such a watch will (i) have to be reset in less than 24 days and (ii) not have to be reset in at least 180 days. (8)

(ii) Messages arrive at a switch board in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events: (i) Exactly two messages arrive within one hour (ii) No messages arrive within one hour (iii) At least three messages arrive within one hour. (8)

17. (a) (i) The joint probability mass function of (X, Y) is given by

$p(x,y) = \frac{1}{72}(2x+3y), x = 0,1,2, y = 1,2,3$. Find all the marginal and conditional probabilities of X and Y. (8)

(ii) The joint PDF of (X, Y) is given by $f(x,y) = e^{-(x+y)}, x, y \geq 0$. Are X and Y independent. (8)

Or

(b) (i) Two random variables X and Y have the joint pdf $f(x,y) = x + y, 0 \leq x, y \leq 1$. Find the probability density function of U= XY. (8)

(ii) Find the correlation coefficient of between X and Y from the data: (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

18. (a) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following cleanness readings were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines:

	Engine 1	Engine 2	Engine 3	Total
Detergent A	45	43	51	139
Detergent B	47	46	52	145
Detergent C	48	50	55	153
Detergent D	42	37	49	128
Total	182	176	207	565

Perform the ANOVA and test at 0.01 level of significance whether there is any differences in the detergents or in the engines. (16)

Or

(b) Analyze the variance in the Latin square of yields (in Kgs) paddy where P, Q, R, S denote the different methods of cultivation

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields. (16)

19. (a) (i) Customers arrive at a watch repair show according to Poisson process at a rate of 1 per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. Compute (i) L_s in the system (ii) W_s in the system (iii) W_q and (iv) the probability that the server is idle. (8)

(ii) A petrol pump station has 4 petrol pumps. The service time follows an exponential distribution with mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. (i) Find the probability that no car is in the system (ii) What is the probability that an arrival will have to wait in the queue (iii) Find the mean waiting time in the system. (8)

Or

(b) (i) An airport has a single runway. Airplanes have been found to arrive at the rate of 15 per hour. It is estimated that each landing takes 3 minutes. Assuming a Poisson process for arrivals and an exponential distribution for landing times. Find the expected number of airplanes waiting to land, expected waiting time. What is the probability that the waiting will be more than 5 minutes. (8)

(ii) Patients arrive at a clinic according to Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour: (i) Find the effective arrival rate at the clinic (ii) What is the probability that an arriving patient will not wait? (iii) What is the expected waiting time until a patient is discharged from the clinic. (8)

20. (a) (i) Write a short note on open queueing network. (8)

(ii) For a 2 stage (service point) sequential queue model with blockage, compute the average number of customers in th system and the average time that a customer has to spend in the system if $\lambda = 1, \mu_1 = 2, \mu_2 = 1$ (8)

Or

(b) Derive Pollaczek-Khintchine formula for the average number of customers in the M/G/1 queueing system. (16)

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