Reg. No. : | | | | | | | | | | |

Question Paper Code: 33001

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2017

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. State the Dirichlet's conditions for the existence of a Fourier series.
2. State the conditions for $f(x)$ to have Fourier series expansion.
3. Find Fourier Sine Transform of $\frac{1}{x}$.
4. Define Fourier Integral theorem.
5. Find the Z-transform of a^n.
6. Find $Z[\frac{1}{n(n+1)}]$.
7. State initial and final value theorems on z - transform.
8. State any two laws which are assumed to derive one dimensional heat equation.
9. Write down the diagonal five point formula in Laplace equation.
10. State Liebmann's iteration process formula.

PART - B (5 x 16 = 80 Marks)

11. (a) Expand the function $f(x) = \sin x, 0 < x < \pi$ in a Fourier cosine series. (16)

Or

(b) Find the Half range cosine series for $y = x$ in *(0, l)* and hence show that

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots \infty .$$ (16)

12\. (a) Find the Fourier cosine and sine transform of $e^{-ax}, a > 0$ and hence evaluate

$$\int_0^\infty \frac{dx}{(a^2 + x^2)^2} \text{ and } \int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$$ (16)

Or

(b) (i) Find the Fourier transform of $e^{-a|x|}$ $if\ a > 0$ (8)

(ii) Evaluate using transform method $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ (8)

13\. (a) Find the inverse z - transform of $\frac{z^2}{(z-a)(z-b)}$ using convolution theorem. (16)

Or

(b) (i) State and prove initial and final value theorem on Z- transform. (8)

(ii) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by using method of Partial fraction. (8)

14\. (a) A rod of length l has its ends A and B maintained at 0° C and 100° C respectively, until steady state conditions prevail. If B is suddenly reduced to 0° C and maintained at 0° C, find the temperature at a distance x from A at time t. (16)

Or

(b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = 3(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at any time t. (16)

15\. (a) Solve $u_{xx} = 32u_t$ for $t \geq 0$, $0 \leq x \leq 1$, $u(0, t) = 0$, $u(x, 0) = 0$ and $u(1, t) = t$ for two time step. (16)

Or

(b) Solve numerically $4u_{xx} = u_{tt}$ with the boundary conditions $u(0,t) = 0$, $u(4,t) = 0$ and the initial conditions $u_t(x,0) = 0$ and $u(x,0) = x\ (4-x)$ taking $h = 1$ up to *4* time steps. (16)