Reg. No. : | | | | | | | | | | |

Question Paper Code: 41301

B.E. / B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The formula for finding the Euler constant a_n of a Fourier series in *[0, 2π]* is ________

   (a) $a_n = \int_0^{\pi} f(x) cosnx\, dx$ (b) $a_n = \int_0^{2\pi} f(x) cosnx\, dx$

   (c) $a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) cosnx\, dx$ (d) $a_n = \frac{1}{\pi} \int_0^{\pi} f(x) cos\frac{n\pi x}{l}\, dx$

2. If *f(x)* is an odd function defined in $(-l\,,\; l)$ the value of a_n is ________

   (a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2n}{\pi}$ (d) 1

3. The Fourier cosine transform of e^{-3x} is ________

   (a) $\frac{2}{\pi}\left(\frac{3}{s^2+3^2}\right)$ (b) $\left(\frac{3}{s^2+3^2}\right)$ (c) $\sqrt{\frac{2}{\pi}}\left(\frac{3}{s^2+3^2}\right)$ (d) $\sqrt{\frac{2}{\pi}}\left(\frac{s}{s^2+3^2}\right)$

4. If Fourier Transform of *f(x) = F(s)* then the Fourier Transform of *f(ax)* is _____

   (a) $\frac{1}{s} F\left(\frac{s}{a}\right)$ (b) $F\left(\frac{s}{a}\right)$ (c) $\frac{1}{2} F\left(\frac{s}{a}\right)$ (d) $\frac{1}{|a|} F\left(\frac{s}{a}\right)$

5. The $Z\{a^n\}$ is ________

(a) $\frac{Z}{Z-a}$, $|Z| > a$ (b) $\frac{Z}{Z-n}$, $|Z| > n$ (c) $a Z- a$ (d) $(Z-a)^n$

6. $Z\{cosn\theta\}$ is ________

(a) $\frac{(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (b) $\frac{Z\,(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (c) $\frac{Z}{Z^2-2Z\,cos\theta+1}$ (d) $\frac{1}{Z^2-2Z\,cos\theta+1}$

7. In the one dimensional wave equation $u_{tt} = a^2 u_{xx}$, where a^2 stands for ______

(a) $1/m$ (b) T (c) $(T/m)^2$ (d) $\frac{T}{m}$

8. The two dimensional Laplace equation is _______

(a) $u_{xx} + u_{yy} = 0$ (b) $u_x + u_y = 0$ (c) $u_{xx} = 0$ (d) $u_{xx} = 4u_{yy}$

9. In the explicit formula for solving one dimensional heat equation, λ is _____

(a) $\frac{a}{h^2}$ (b) ah (c) $\frac{k}{ah^2}$ (d) ka

10. Liebmann's iteration process is used to solve the _________

(a) One dimensional heat flow equation
(b) Two dimensional heat flow under steady state equation
(c) One dimensional wave equation
(d) Hyperbolic equation

PART - B (5 x 2 = 10 Marks)

11. Find the Fourier constants b_n for $x\ sinx$ in $(-\pi, \pi)$.

12. Find the Fourier sine transform of $\frac{1}{x}$.

13. Find the Z transform of $x(n) = \begin{cases} \frac{a^n}{n!} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

14. Classify $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8\,u_y - 16u = 0$.

15. State Standard Five Point Formula (SFPF) and Diagonal Five Point Formula (DFPF).

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Express $f(x) = (\pi - x)^2$ as a Fourier series of periodicity 2π in $0 < x < 2\pi$. (8)

(ii) Find the Fourier series for $f(x) = x^2$ in $-\pi , x < \pi$. (8)

Or

(b) (i) Find the Fourier series of periodicity 3 for $f(x) = 2x - x^2, 0 < x < 3$. (8)

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ given by the following table: (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

17. (a) Find the Fourier Transform of $f(x) = f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$

Hence show that (1) $\int_0^\infty \frac{sins - s\ coss}{s^3} cos\frac{s}{2}\ ds = \frac{3\pi}{16}$

(2) $\int_0^\infty \frac{(xcosx - sinx)^2}{x^6}\ dx = \frac{\pi}{15}$ (16)

Or

(b) (i) Show that Fourier Transform of $f(x) = e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(4+x^2)\ (25+x^2)}$ using Fourier transform method. (8)

18. (a) (i) (1) State and prove the linear property on Z-Transform. (4)

(2) State the initial and final value theorems on Z-Transform. (4)

(ii) Find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z-1)}\right)$ by convolution method. (8)

Or

(b) (i) Solve by Z-transforms $y(n+2) + 6y(n+1) + 9y(n) = 2^n$ given $y(0) = y(1) = 0$. (8)

(ii) Using convolution theorem, find the inverse Z-Transform of $\frac{z^2}{(z-1)(z-3)}$. (8)

19. (a) A tightly stretched string with fixed end points $x = 0$ and $x = 10$ is initially in a position Given by $y(x, 0) = k(10x - x^2)$. It is released from rest from this position. Find the expression for the displacement at any time 't'. (16)

Or

(b) A rectangular plate with insulated surface is $10cm$ wide and so long compared to its width that it may be considered infinite length without introducing appreciable error. If the temperature at short edge $y = 0$ is given by $u = \begin{cases} 20x, & 0 \le x \le 5 \\ 20(10 - x), & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at 0^0C. Find the steady-state temperature at any point of the plate. (16)

20. (a) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units; satisfying the following boundary conditions:

(i) $u(0, y) = 0$ for $0 \le y \le 4$

(ii) $u(4, y) = 12 + y$ for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$ for $0 \le x \le 4$

(iv) $u(x, 4) = x^2$ for $0 \le x \le 4$ (16)

Or

(b) (i) Solve by Bender Schmidt method $u_{xx} = 2u_t$ given $u(0, t) = 0$, $u(4, t) = 0$, $u(x,0) = x(4-x)$ assuming $h=k=1$. Find the values of u up to $t =5$. (8)

(ii) Solve by Crank-Nicholson method the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to $u(x ,0) = 0$, $u(0, t) = 0$ and $u(1, t) = t$ for single step when $h=1/4$, $k= 1/16$. (8)