Reg. No. : | | | | | | | | | | |

Question Paper Code: 31444

B.E/B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Find *c*, if a continuous random variable *X* has the density function $f(x) = \dfrac{c}{1+x^2}, -\infty \leq x \leq \infty$.

2. The moment generating function of a random variable *X* is given by $(t) = e^{2(e^t-1)}$. What is *P [X =0]*?

3. The regression equations are *3X +2Y = 26* and *6X + Y = 31*. Find the correlation coefficient between *X* and *Y*.

4. The joint p.d.f of the *RV (X,Y)* is given by $f(x,y) = kxye^{-(x^2+y^2)}, x > 0 , y > 0$. Find the value of *k*.

5. Define a Markov chain and give an example.

6. State the postulates of a Poisson process.

7. Find the mean and variance of the stationary process {x (t)} whose auto correlation function is given by $(\tau) = 25 + \frac{4}{1+6\tau^2}$.

8. Prove that for a WSS process {X (*t*)}, $R_{XX}(t, t+\tau)$ is an even function of τ.

9. Define white noise.

10. Define average power in the response of a linear system.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find the value of k, $P(1.5 < X < 4.5/ X > 2)$ and the smallest value of k for which $P(X \leq k) > ½$. (8)

(ii) The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$.

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours? (8)

Or

(b) (i) Describe Binomial distribution. Obtain its moment generating function. Hence compute its mean and variance. (8)

(ii) The life of a certain kind of electronic device has a mean of 300 hours and standard deviation of 25 hours. Assuming that the life times of the devices follow normal distribution.

(1) Find the probability that any one of these devices will have a life time more than 350 hours.

(2) What percentage will have life time between 220 and 260 hours? (8)

12. (a) (i) The joint probability mass function of $p(X, Y) = k(2x + 3y)$, x = 0, 1, 2; y = 1, 2, 3. Find k and all the marginal and conditional probability distributions. Also find the probability distribution of $(X + Y)$. (8)

(ii) The joint probability density function of the two dimensional random variable (X, Y) is $(x, y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & , else \text{ where} \end{cases}$. Find the correlation coefficient between X and Y. (8)

Or

(b) (i) If X and Y are independent RVs each normally distributed with mean zero and variance σ^2, find the p.d.f of $R = \sqrt{X^2 + Y^2}$ and $\varphi = \tan^{-1}\left(\frac{Y}{X}\right)$. (16)

13. (a) (i) Examine whether the random process $\{X(t)\} = A \cos(\omega t + \theta)$ is a wide sense stationary if A and ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) Prove that the interval between two successive occurrences of a Poisson process with parameter λ has an exponential distribution with mean $1/\lambda$. (8)

Or

(b) (i) Define random telegraph signal process. Prove

(a) $P[X(t) = 1] = 1/2 = P[X(t) = -1]$, for all $t > 0$

(b) $E[X(t)] = 0$ and $Var[X(t)] = 1$ (8)

(ii) A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find

(1) the probability that he takes a train on the third day

(2) the probability that he drives to work in the long run. (8)

14. (a) State and prove Weiner – Khintchine Theorem. (16)

Or

(b) (i) The cross – power spectrum of real random processes $\{X(t)\}$ and $\{Y(t)\}$ is given

$S_{xy} = \begin{cases} a + bi\omega, & for\ |\omega| \le 1 \\ 0, & elsewhere \end{cases}$. Find the cross correlation function. (8)

(ii) If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively then prove that $|R_{XY}(\tau)| \le \sqrt{R_{XY}(0)R_{YY}(0)}$. (8)

15. (a) (i) Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output $\{Y(t)\}$ is a WSS process. (8)

(ii) Let $X(t)$ be a WSS process which is the input to a linear time invariant system with unit impulse h(t) and output $Y(t)$, then prove that where $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$ where, $H(\omega)$ is Fourier transform of $h(t)$. (8)

Or

(b) If $(t) = A\cos(\omega_0 t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band-limited Gaussian white noise with power spectral density $S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & \text{for } |\omega - \omega_0| \leq \omega_B \\ 0, & elsewhere \end{cases}$. Find the power spectral density $\{Y(t)\}$. Assume that $\{N(t)\}$ and θ are independent. (16)