Reg. No. :

Question Paper Code: 31401

B.E. / B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Fourth Semester

Computer Science and Engineering

01UMA421 - APPLIED STATISTICS AND QUEUING NETWORKS

(Common to Information Technology)

(Regulation 2013)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

(Use of statistical tables may be permitted)

PART A - (10 x 2 = 20 Marks)

1. If X is a Poisson variate such that $2P[X = 0] + P[X = 1] = 2P[X = 1]$, find $E[X]$.

2. If X is uniformly distributed over the interval (0, 10), find (a) $P(X>6)$, (b) $P(3 < x < 8)$.

3. Two random variables X and Y have PDF $f(x, y) = k(2x+y)$ for $0 \leq x \leq 2, 0 \leq y \leq 3$, solve for k.

4. State central limit theorem.

5. What is the aim of the design of experiments?

6. Compare completely randomized experimental design and latin square design.

7. In a 3 server infinite capacity Poisson queue model if $\frac{\lambda}{s\mu} = \frac{2}{3}$, find P_0.

8. State Little's formula for $(M/M/1):(k/FIFO)$ model.

9. For an $M/G/1$ model if $\lambda = 5min, \mu = 6min$ and $\sigma = \frac{1}{20}$, find the length of the queue.

10. Give the flow balance equations for an open Jackson network.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A company has two plants to manufacture scooters. Plant I manufactures 80% of the scooters and plant II the rest. AT plant I, 85 out of 100 scooters are rated higher quality and at plant II, only 65 out of 100 scooters are rated higher quality. A scooter is chosen at random. What is the probability that the scooter came from Plant II, if it is known that the scooter is of higher quality. (8)

(ii) If 10% of the screws produced by an automatic machine are defective, find the probability that of 20 screws selected at random, there are (1) exactly two defectives (2) atmost three defectives (3) atleast two defectives. (8)

Or

(b) (i) Let the probability density function for X be given by

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

Find (1) Moment generating function for X.
(2) $P(X > 3)$
(3) E(X) +
(4) Var(X). (8)

(ii) An electrical firm manufactures light bulbs that have a life, before burn-out that is normally distributed with mean equal to 800 hours and standard deviation of 40 hours. Estimate

(1) the probability that a bulb burns more than 834 hours
(2) the probability bulb burns between 778 and 834 hours. (8)

12. (a) The joint pdf of a two-dimensional random variable (X, Y) is given by

$$f(x,y) = \left(xy^2 + \frac{x^2}{8}\right); \ 0 \leq x \leq 2, 0 \leq y \leq 1. \quad \text{Compute}$$

(i) $P(X>1/Y<1/2)$ (ii) $P(Y<1/2/X>1$ (iii) $P(X<Y)$ (iv) $P(X+Y \leq 1)$. (16)

Or

(b) (i) Calculate the correlation coefficient for, the following data: (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

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(ii) Let X and Y be dimensional random variables having the joint probability density function is given by

$$f(x,y) = \begin{cases} e^{-(x+y)} \text{ for } x>0, \ y>0 \\ 0, \ otherwise \end{cases}. \text{ Find the pdf of } U = \frac{X}{X+Y}. \hspace{1cm} (8)$$

13. (a) Five breeds of cattle B_1, B_2, B_3, B_4, B_5 were fed on four different rations R_1, R_2, R_3, R_4. Gains in weight in kg. over a given period were recorded and given below:

	B_1	B_2	B_3	B_4	B_5
R_1	1.9	2.2	2.6	1.8	2.1
R_2	2.5	1.9	2.3	2.6	2.2
R_3	1.7	1.9	2.2	2.0	2.1
R_4	2.1	1.8	2.5	2.3	2.4

Is there a significant difference between (i) breeds and (ii) rations? (16)

Or

(b) Analyze the variance in the following latin square of yield (*in kgs*) of paddy where *A, B ,C, D* denote the different methods of cultivation

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields. (16)

14. (a) Customer arrive at a one-man barber shop according to a Poisson process with a mean interarrival time of 12 *min.* Customers spend an average of 10 *min* in the barber's chair.

    (i) What is the expected number of customers in the barber shop and in the queue?

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(ii) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.

(iii) How much time customers expect to spend in the barber's shop?

(iv) What is the average time customer spends in the queue?

(v) What is the probability that more than 3 customers are in the system?

(vi) What is the probability that the waiting time the system is greater than 30 *min*?

(16)

Or

(b) A car servicing station has 2 bays where service can be offered simultaneously due to space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with a mean of 12 cars per bay. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Estimate

(i) The average number of cars in the service station

(ii) The average number of cars waiting for service

(iii) The average time a car spends in the system. (16)

15. (a) State and prove Pollaczek-Khintchine formula. (16)

Or

(b) A TVS company in chennai containing a repair section shared by large number of machines has 2 sequential stations with respective service rates of 3 per hour and 4 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behavior can be approximated by the above 2 stage tendon queue, calculate

(i) the probability that both service stations are idle

(ii) the average number of machines in the repair section.

(iii) the average repair time including the waiting time.

(iv) the bottleneck of the repair facility. (16)

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