Reg. No. : | | | | | | | | | | |

Question Paper Code: 31301

B.E. / B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Third Semester

Civil Engineering

01UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. State the Dirichlet's conditions for the existence of a Fourier series.

2. If the Fourier series of the function $f(x) = x + x^2$ in the interval $-\pi \leq x \leq \pi$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty}(-1)^n\left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$

3. Prove that if $F(s)$ is the Fourier transform of $f(x)$, then $F\{ f(x - a) \} = e^{isa} F(s)$.

4. Find Fourier cosine transform of $e^{-ax} \cos ax$.

5. Find Z transform of a^n.

6. State convolution theorem of Z–transform.

7. State any two laws assumed to derive the one dimensional wave equation.

8. When the ends of a rod length $20cm$ are maintained at the temperature $10^{o}c$ and $20^{o}c$ respectively until steady state is prevail. Determine the steady state temperature of the rod.

9. Obtain the finite difference scheme for the difference equations $2\frac{d^2y}{dx^2} + y = 5$.

10. Write down the diagonal five point formula in Laplace equation?

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Obtain the Fourier series for $f(x) = (l-x)^2 \ in \ (0, 2l)$ and hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\dots\dots=\frac{\pi^2}{6}$. (8)

(ii) Find the half range cosine series of $f(x) = xsinx \ in (0, \pi)$. (8)

Or

(b) (i) Find the half-range sine series of $f(x) = l - x \ in \ (0, l)$ and hence find the value of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ (8)

(ii) Find the Fourier series expansion of period 2π for the function $y = f$ (x) which is defined in *(0, 2π)* by means of the table value given below. Find the series up to second harmonic. (8)

X	*0*	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
Y	*1.0*	*1.4*	*1.9*	*1.7*	*1.5*	*1.2*	*1.0*

12. (a) (i) Find the Fourier transform of $f(x)$ is defined as $f(x) = \begin{cases} a^2 - x^2 & \text{if } |x| < a \\ 0 & otherwise \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt$. (8)

(ii) Show that $e^{-a^2x^2}$ is self – reciprocal with respect to Fourier transform, when $a = \frac{1}{\sqrt{2}}$. (8)

Or

(b) (i) Use Fourier transform technique to evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ (8)

(ii) Find the Fourier cosine transform of the function $f(x) = \frac{e^{-ax}}{x}$ (8)

13. (a) (i) Using convolution theorem, find $z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ (8)

(ii) Find the z – transform of $\frac{1}{(n+1)(n+2)}$ (8)

Or

(b) (i) Solve $u_{n+2} - 5u_{n+1} + u_n = 4^n$ given that $u_0 = 0,\ u_1 = 1$ using Z – transform. (8)

(ii) Find the inverse Z-transform of $\frac{z^3-20z}{(z-2)^3(z-4)}$. (8)

14. (a) A tightly stretched string of length l has its ends fastened at x = 0, x = l. The mid-point of the string is then taken to height h and then released from rest in that position. Find the lateral displacement of a point of the string at time t from the instant of release. (16)

Or

(b) The ends A and B of a rod l cm long have the temperature at 30^0c and 80^0c until steady state prevails. The temperature of the ends is then changed to 40^0c and 60^0c respectively. Find the temperature distribution in the rod at any time. (16)

15. (a) Solve $u_{xx} + u_{yy} = 0$ over the square mesh of side 4 units; satisfying the following boundary conditions:

(i) $u(0,y) = 0\ for\ 0 \le y \le 4$

(ii) $u(4,y) = 12 + y\ \ for\ 0 \le y \le 4$

(iii) $u(x,0) = 3x\ for\ 0 \le x \le 4$

(iv) $u(x,4) = x^2\ for\ 0 \le x \le 4$ (16)

Or

(b) (i) Solve $u_{xx} = 32u_t$ for $t \ge 0$, $0 \le x \le 1$, $u(0, t) = 0$, $u(x, 0) = 0$ and $u(1, t) = t$ for two time step. (8)

` (ii) Solve $u_t = u_{xx}$ given $u(0, t) = 0$ and $u(4, t) = 0$, $u(x, 0) = x(4-x)$ assuming $h=k=1$. Find the values of u up to $t=5$. (8)