Reg. No. : | | | | | | | | | | |

Question Paper Code: 31202

B.E. / B.Tech. DEGREE EXAMINATION, NOVEMBER 2015

Second Semester

Civil Engineering

01UMA202 - ENGINEERING MATHEMATICS - II

(Common to ALL branches)

(Regulation 2013)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Solve $(x^2 D^2 + xD - 1)\, y = 0$.

2. Transform the equation $(x^2 D^2 + xD) y = x$ into a linear differential equation with constant coefficients.

3. Find the directional derivative of $\phi = x^2 yz + 4xz^2 + xyz$ at *(1, 2, 3)* in the direction of $2\vec{i} + \vec{j} - \vec{k}$.

4. Find 'a' such that $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$ is solenoidal.

5. Test the analyticity of the function $f(z) = \bar{z}$.

6. Find the fixed points of $w = \dfrac{3z - 4}{z - 1}$.

7. State Cauchy's integral formula for first derivative of an analytic function.

8. Expand $\dfrac{1}{z - 2}$ at $z = 1$ in a Taylor's series.

9. State and prove the shifting property in Laplace Transform.

10. Define existence conditions of Laplace transform.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Solve: $(2x + 3)^2 \frac{d^2y}{dx^2}$ - $2(2x + 3) \frac{dy}{dx}$ - 12y = 6x. (8)

(ii) Solve $(D^2+4)\, y = x \sin x$. (8)

Or

(b) (i) Solve $(D^2 - 4D + 4)y = \frac{e^{2x}}{x}$ by the method of variation of parameters. (8)

(ii) Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2 + \cos(\log x)$. (8)

12. (a)(i) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\vec{i} + 2\,\vec{j} + 2\,\vec{k}$. (8)

(ii) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\,\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F} = \Delta\phi$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = xz\,\vec{i} + 4xy\vec{j} - z^2\vec{k}$ over the cube bounded by $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$. (16)

13. (a) (i) If $f(z)$ = u + iv is a regular function of z, then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) | f(z)|^p = p^2 \,|\, f(z)|^{p-2} | f'(z)|^2$. (8)

(ii) Find the bilinear transformation which maps x = 1, i , -1 respectively onto w = i, 0, -i. . (8)

Or

(b) (i) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$. (8)

14. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z+1)(z+2)} dz$ where C is the circle $|z| = 3$. (8)

(ii) Using Residue theorem evaluate $\int_C \frac{z}{(z-1)(z-2)} dz$ where C is the circle $|z - 2| = 1/2$. (8)

Or

(b) (i) Find the Laurent's series of $f(z) = \frac{7z - 2}{z(z+1)(z-2)}$ in $1 < |z+1| < 3$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$. (8)

15. (a) (i) Find the Laplace transform of the function $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ such that $f(t + 2a) = f(t)$. (8)

(ii) Find $L^{-1}\left[\frac{s}{(s+1)(s+2)}\right]$. (8)

Or

(b) (i) Find the Laplace transform of the Half-wave rectifier function

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}, \text{with } f\left(t + \frac{2\pi}{\omega}\right) = f(t).$$ (8)

(ii) Using convolution theorem find $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$. (8)