Reg. No. : | | | | | | | | | | |

Question Paper Code: 51102

B.E. / B.Tech. DEGREE EXAMINATION, DECEMBER 2015

First Semester

Civil Engineering

15UMA102 - ENGINEERING MATHEMATICS – I

(Common to ALL branches)

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $d\left(x^2 \log x\right)$ with respect to 'x' is _____

(a) $x\left(1 + 2\log x\right)$ (b) $\left(x + 2\log x\right)$ (c) $x\left(x + 2\log x\right)$ (d) $\left(1 + 2\log x\right)$

2. Identify the value of $\frac{d^3}{dx^3}\left(e^{3x}\right)$ at $x = 0$.

(a) 1 (b) 3 (c) 6 (d) 9

3. If $x = r\cos\theta$ and $y = r\sin\theta$, then Jacobin of r and θ is

(a) $\frac{1}{r}$ (b) r (c) $r\sin\theta$ (d) $r^2\sin\theta$

4. If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(a) 0 (b) nu (c) $r\sin 2u$ (d) $\sin 2u$

5. If *f(x)* is odd function, then $\int_{-a}^{a} f(x)\,dx$ is equal to

(a) $2\int_{0}^{a} f(x)\,dx$ (b) $2f(a)$ (c) 0 (d) 1

6. $\int_{0}^{\infty} \frac{e^{-x}}{\sqrt{x}}\,dx$ is equal to

(a) $\sqrt{\pi}$ (b) $\sqrt{\frac{1}{2}}$ (c) $\frac{\sqrt{\pi}}{2}$ (d) $\frac{2}{\sqrt{\pi}}$

7. $\int_{1}^{2}\int_{1}^{3} \frac{dxdy}{xy}$ is equal to

(a) $\log 2\ \log 3$ (b) $\log 2 + \log 3$ (c) $2\log 6$ (d) 2

8. $\int_{0}^{1}\int_{0}^{2}\int_{0}^{3} xydxdydz$ is equal to

(a) 3 (b) 6 (c) 12 (d) 36

9. If product of two Eigen value is 12 and determinant is 36 of the 3 x 3 matrix, then find the third Eigen value.

(a) 2 (b) 3 (c) 4 (d) 5

10. If $X = (1, m, 3)^T$ and $Y = (6, 3, 1)^T$ are orthogonal vector, then find the value of *m*.

(a) 0 (b) 1 (c) -1 (d) -3

PART - B (5 x 2 = 10 Marks)

11. Differentiate $\sin(\tan x)$.

12. Write the any two properties of Jacobin.

13. Prove that Beta function satisfies commutative law.

14. Change the order of integration $\int_{0}^{2}\int_{0}^{y} dxdy$.

15. Find the nature of the $2x^2 - 3y^2 + z^2$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the n^{th} derivative of $\frac{1}{x^2 + a^2}$. (8)

(ii) A body originally at 80°*C* cools down to 60°*C* in 20 minutes, the temperature of the air being 40°*C*. What will be the temperature to the body after 40 minutes from the original? (8)

Or

(b) (i) Expand $e^{\sin x}$ using Maclaurin's series. (8)

(ii) Prove that $\left(x^2 - 1\right) y_{n+2} + \left(2n + 1\right) x y_{n+1} + \left(n^2 - m^2\right) y_n = 0.$

Given that $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$ using Leibnitz's theorem. (8)

17. (a) (i) Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$ where $\log u = \frac{\left(x^3 + y^3\right)}{\left(3x + 4y\right)}$. (8)

(ii) Find the longest and shortest distances of the point (1, 2, -1) from the sphere $x^2 + y^2 + z^2 = 24.$ (8)

Or

(b) (i) Expand $e^x \log\left(1 + y\right)$ using Taylor's expansion. (8)

(ii) A rectangle box open at the top is to have volume of 32 cubic ft. Find the dimension of the box. (8)

18. (a) (i) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx.$ (8)

(ii) Prove that $\int_0^1 \frac{x^2\, dx}{\sqrt{\left(1 - x^4\right)}} \times \int_0^1 \frac{dx}{\sqrt{1 - x^4}} = \frac{\pi}{4}.$ (8)

Or

(b) (i) Evaluate $\int \cos^7 x dx$ using reduction formula. (8)

(ii) Prove that $\int_0^{\infty} \frac{x^5}{5^x} dx = \frac{120}{\left(\log 5\right)^6}.$ (8)

19. (a) (i) Find the volume bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the cone $x^2 + y^2 = z^2$. (8)

(ii) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2 - x^2 - y^2}\, dxdy$. (8)

Or

(b) (i) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. (8)

(ii) Change the order of integration and hence evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$. (8)

20. (a) (i) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (8)

(ii) Verify Cayley – Hamilton theorem and hence find A^{-1} if $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (8)

Or

(b) Reduce $8x^2 + 7y^3 + 3z^3 - 12xy - 8yz + 4zx$ into canonical form by an orthogonal reduction and find the rank, index, signature and the nature of the quadratic form. (16)

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