Reg. No. : | | | | | | | | | | |

Question Paper Code: 52185

M.E. DEGREE EXAMINATION, DECEMBER 2015

First Semester

Structural Engineering

15PMA125 - APPLIED MATHEMATICS FOR STRUCTURAL ENGINEERING

(Regulation 2015)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions

PART A - (5 x 1 = 5 Marks)

1. $F(e^{-x^2/2}) =$ ______________

   (a) $e^{s^2/2}$ (b) $e^{-x^2/2}$ (c) $e^{-s^2/2}$ (d) $e^{x^2/2}$

2. For one point Gaussian Quadrature the sampling point is at _____________

   (a) $\xi = 0$ (b) $\xi = 2$ (c) $\xi = 3$ (d) $\xi = 1$

3. Suppose 'f' is independent of 'y' then the solution of Euler's equation is ______________

   (a) $\frac{\partial F}{\partial y'} = c$ (b) $\frac{\partial F}{\partial y} = c$ (c) $\frac{\partial F}{\partial x} = c$ (d) $\frac{\partial F}{\partial x'} = c$

4. To find the dominant eigen value of a matrix then use ________________

   (a) Approximation method (b) Power method
   (c) Rayley-Ritz method (d) Faddeev-Leverrier method

5. Angle between the regression lines are parallel then ________________

   (a) $\theta = 0$ (b) $\theta = \frac{\pi}{2}$ (c) $\theta = \frac{\pi}{4}$ (d) $\theta = \pi$

PART - B (5 x 3 = 15 Marks)

6. If $u(x, t)$ is a function of two variables x and t, prove that

$$L\left[\frac{\partial u}{\partial x}; s\right] = \frac{dU(x,s)}{dx}$$

7. Define Rayleigh quotient of a Hermitian matrix.

8. Obtain the Euler's equation for the extremals of the functional $\int_{x_0}^{x_1} \left(y^2 - yy' + y'^2\right) dx$.

9. Find the largest eigen value of $\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ by Power method.

10. What are maximum likelihood estimators?

PART - C (5 x 16 = 80 Marks)

11. (a) Using the Laplace transform method, solve the IBVP described as

PDE: $u_{xx} = \frac{1}{c^2} u_{tt} - \cos \omega t, \quad 0 \le x < \infty, \quad 0 \le t < \infty$

BCs: $u(0, t) = 0$, u is bounded as x tends to ∞

ICs: $u_t(x, 0) = u(x, 0) = 0.$ (16)

Or

(b) Solve the following IBVP using the Laplace transform technique

PDE : $u_t = u_{xx}, \quad 0 < x < 1, \quad t > 0$

BCs : $u(0, t) = 1, u(1, t) = 1, t > 0$

ICs : $u(x, 0) = 1 + \sin \pi x, 0 < x < 1.$ (16)

12. (a) (i) By relaxation method, solve $12x + y + z = 31,\ 2x + 8y - z = 24,\ 3x + 4y + 10z = 58.$ (8)

(ii) Solve the equation by Choleski method
$4x + 6y + 8z = 0,\ 6x + 34y + 52z = -160,\ 8x + 52y + 129z = -452.$ (8)

Or

(b) (i) Evaluate $\int_1^2 \frac{dx}{1+x^3}$ by Gaussian three point formula. (8)

(ii) Evaluate $\int_1^2\int_1^2 \frac{dxdy}{x+y}$ by Gaussian quadrature formula. (8)

13. (a) (i) By applying Ritz method, find the extremal of $I[y(x)] = \int_0^1 (y'^2 + y^2)dx$ with $y(0) = 0, y(1) = 1$. (8)

(ii) Find the plane curve of a fixed perimeter and maximum area. (8)

Or

(b) (i) Find the extremal of the functional $I[y(x)] = \int_{-a}^{a} \left(\frac{1}{2}\mu y''^2 + \rho y\right)dx$ that satisfies the boundary conditions $y(-a) = 0, y'(-a) = 0, y(a) = 0, y'(a) = 0.$ (8)

(ii) Prove that the sphere is the solid figure of a revolution which for a given surface has maximum volume. (8)

14. (a) Using power method find all the Eigen values of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (16)

Or

(b) Use Faddeev-Leverrier method to find the characteristic polynomial and inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$. (16)

15. (a) (i) Fit a parabola $y = a + bx + cx^2$ to the following data by the method of least squares

X :	2	4	6	8	10
Y :	3.07	12.85	31.47	57.38	91.29

(8)

(ii) Estimate α and β for the distribution defined by

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{I(x)}x^{\alpha-1}e^{-\beta x},\ 0\le x\le\infty$$ by the method of moments. (8)

Or

(b) (i) In a trivariate distribution r_{12} = 0.7, r_{13} = r_{23} = 0.5. Find the partial correlation coefficient $r_{12.3}$ and multiple correlation coefficients $R_{1.23}$. (8)

(ii) In a random sampling from normal population $N(\mu,\sigma^2)$, find the maximum likelihood estimators for μ when σ^2 is known. (8)