Reg. No. : | | | | | | | | | | |

Question Paper Code: 52184

M.E. DEGREE EXAMINATION, DECEMBER 2015

First Semester

CAD / CAM

15PMA124 - ADVANCED NUMERICAL METHODS

(Regulation 2015)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (5 x 1 = 5 Marks)

1. In calculating $\sqrt{N}$, the iterative formula is

   (a) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ (b) $x_{n+1} = \frac{1}{4}\left[x_n + \frac{N}{x_n}\right]$

   (c) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{x_n}{N}\right]$ (d) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{x_n}{N^2}\right]$

2. In second order R-K method, the value of $y\ (0.1)$ given $y' = -y, y(0) = 1$ is

   (a) 0.705 (b) 0.905 (c) 1.101 (d) 0.606

3. In solving $u_t = \alpha^2 u_{xx}$, by Crank-Nicholson scheme, we take $\frac{(\Delta x)^2}{\alpha^2 k}$ as

   (a) 2 (b) 3 (c) 1 (d) 4

4. Classify the equation $u_{xx} + 2u_{xy} + 4u_{yy} = 0$

   (a) parabolic (b) cycloidal (c) hyperbolic (d) elliptic

5. The error in the diagonal formula is ____ times the error in the standard formula.

   (a) 1 (b) 2 (c) 3 (d) 4

PART - B (5 x 3 = 15 Marks)

6. Derive the criterion for convergence in Newton- Raphson Method.

7. Write down the algorithm of Runge-Kutta method of fourth order.

8. State the implicit scheme to solve one dimensional heat equation numerically.

9. Write down the Diagonal scheme to solve the Laplace equation $u_{xx} + u_{yy} = 0$.

10. What is Galerkin Finite element method?

PART - C (5 x 16 = 80 Marks)

11. (a) (i) Solve by Gauss-Seidel method $x + y + 54z = 110,\ 27x + 6y - z = 85,$ $6x + 15y + 2z = 72,$ correct to three decimal places. (8)

(ii) Find a positive root of $xe^x = 1$ by Newton's method. (8)

Or

(b) (i) Solve by Gauss Jordan method $10x + y + z = 12,\ 2x + 10y + z = 13,$ $x + y + 5z = 7.$ (8)

(ii) Find the largest Eigen value of the matrix $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and corresponding Eigen vector by using power method. (8)

12. (a) (i) Given $y'' + xy' + y = 0, y(0) = 1, y'(0) = 0$. Find the value of $y(0.1)$ by using Runge-Kutta method of fourth order. (8)

(ii) Solve the ODE: $y'' - 64y + 10 = 0$ with $y(0) = 0 = y(1)$ and $h = \frac{1}{4}$. (8)

Or

(b) Using Adam's Bashforth modified method, find y at $x = 1.4$ given that $\frac{dy}{dx} = x^2(1 + y)$ and $y(1) = 1, y(1.1) = 1.233.\ y(1.2) = 1.548$ and $y(1.3) = 1.979.$ (16)

13. (a) (i) Given $u_t = u_{xx}$, subject to $u(x,0) = \frac{x}{3}(16 - x^2);\ u(0,t) = 0 = u(4,t).$ Find u_{ij}, i = 1, 2, 3, 4 and j = 1, 2 by Nicolson's method. (8)

(ii) Solve $u_{tt} = 4u_{xx}, 0 < x < 10, t > 0$ satisfying $u(10,t) = 0,$ $u_t(x,0) = \frac{1}{100}x(10 - x), 0 \le x \le 10.$ Compute u for three time steps. (8)

Or

(b) (i) Solve the equation $u_t = u_{xx}$
subject to $u(x,0) = sin\pi x; 0 \le x \le 1;\ u(0,t) = 0 = u(1,t)$ by Schmidt method. (8)

(ii) Solve $25u_{xx} - u_{tt} = 0$. Given that $u(0,t) = 0 = u(5,t)$, $u(x,0) = 0$,

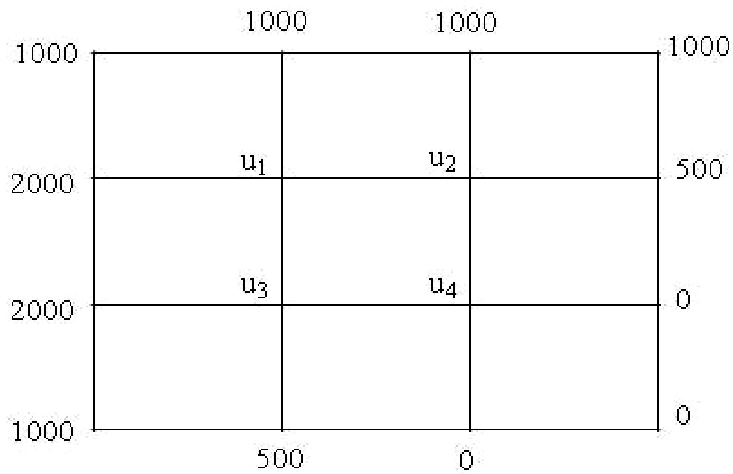
$$u(x,0) = \begin{cases} 2x, 0 < x < 2.5 \\ 10 - 2x,\ 2.5 < x < 5 \end{cases}$$ for one period with $h = 1$. (8)

14. (a) Solve $u_{xx} + u_{yy} = 0, 0 \le x \le 4, 0 \le y \le 4$. Given that $u(0,y) = 0$,

$u(4,y) = 8 + 2y,\ u(x,0) = \frac{x^2}{2}$ and $u(x,4) = x^2$ taking $h = k = 1$. (16)

Or

(b) Evaluate the function $u(x,y)$ satisfying the Laplace equation such that (16)



15. (a) Solve the Poisson's differential equation $\nabla^2 u = -4(x^2 + y^2)$, with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$, with $u = 0$ on the boundary and mesh length 1 unit. (16)

Or

(b) Solve the boundary value problem $u_{xx} + u_{yy} = -2, |x| \le 2,\ |y| \le 2$ and $u = 0$ on the boundary. Use the Galerkin finite element method to determine u at the nodes (0, 0), (1,0) and (1, 1) (16)