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Question Paper Code : 33539

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fourth Semester

Information Technology

MA 1253/MA 1259 — PROBABILITY AND STATISTICS

(Common to Production Engineering, Automobile Engineering, Mechanical Engineering, Information Technology, Textile Technology, Textile Technology (Textile Chemistry) and Textile Technology (Fashion Technology)

(Also common to Sixth Semester – Civil Engineering)

(Regulation 2004/2007)

(Common to B.E. (Part-Time) Third Semester, Mechanical Engineering – Regulation 2005)

Time : Three hours | Maximum : 100 marks

Use of approved Statistical Table is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $P[E \cup F] = P[E] + P[F] - P[E \cap F]$ where E and F are any two events.

2. Find $E[X]$ when the density function of X is $f(x) = \begin{cases} 2x & if\ 0 \le x \le 1 \\ 0 & otherwise \end{cases}$

3. Define Weibull distribution function.

4. If X is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$, find $P\{|X-3| > 6\}$.

5. Suppose that $p(x, y)$, the joint probability mass function of X and Y, is given by $p(0, 0) = 0.4$, $p(0, 1) = 0.2$, $p(1, 0) = 0.1$, $p(1, 1) = 0.3$. Calculate the conditional probability mass function of X, given that $Y = 1$.

6. State Central Limit Theorem.

7. Test the null hypothesis $H_0 : \mu = 180$ if $H_1 : \mu < 180$, $\alpha = 0.01$, $n = 5$ and $\bar{x} = 169.5$, $s = 5.7$.

8. Suppose that we want to estimate the true proportion of defectives in a very large shipment of adobe bricks and that we want to be atleast 95% confident that the error is at most 0.04. How large a sample will we need if we know that the true proportion does not exceed 0.12?

9. Explain briefly about a randomized-block design.

10. Define Latin square design.

PART B — (5 × 16 = 80 marks)

11. (a) (i) In a certain assembly plant, three machines B_1, B_2 and B_3, make 30%, 45% and 25% respectively of the products. It is known from past experience that 2%, 3% and 2% of the products made by each machine respectively are defective. Suppose that a finished product is randomly selected, what is the probability that it is defective? (8)

(ii) Calculate variance of X if X represents the outcome when a fair die is rolled. (8)

Or

(b) (i) Find the moment generating function of a random variable X whose probability density function is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$, $-\infty < x < \infty$. (10)

(ii) The weekly demand for a certain drink, in thousands of liters, at a chair of convenience stores is a continuous random variable.

$g(X) = X^2 + X - 2$, where X has the density function
$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & elsewhere \end{cases}$$

Find the expected value for the weekly demand of the drink. (6)

12. (a) (i) The mean and standard deviation of marks in Mathematics are 70 and 10 respectively. The corresponding values for control systems are 55 and 15 respectively. Assume that the marks in the two subjects are independent normal variates, find the probability that a student scores a total of marks lying between 100 and 120 in the two subjects? (10)

(ii) Derive the mean and variance of Gamma distribution. (6)

Or

(b) (i) Find the mean and variance of Weibull distribution. (10)

(ii) In a certain industrial facility accidents occur in frequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. What is the probability that is any given period of 400 days there will be an accident on one day? (6)

13. (a) If the joint probability density function of two random variables X and Y is given by $f(x, y) = \frac{6 - x - y}{8}$, $0 \le x \le 2$, $2 \le y \le 4$ find the correlation coefficient between X and Y.

Or

(b) If X and Y are independent random variables with probability density functions

$f_X(x) = \alpha e^{-\alpha x}$, $x > 0$; $f_Y(y) = \beta e^{-\beta y}$, $y > 0$ find the pdf of $X - Y$.

14. (a) To compare two kinds of bumper guards, 6 of each kind were mounted on a certain kind of compact car. Then each car was run into a concrete wall at 5 miles per hour and the following are the costs of the repairs (in dollars) :

Bumper guard 1 :	107	148	123	165	102	119
Bumper guard 2 :	134	115	112	151	133	129

Use 0.01 level of significance to test whether the difference between the two sample means is significant.

Or

(b) The following is the distribution of the daily number of power failures reported in a western city on 300 days :

No. of power failures :	0	1	2	3	4	5	6	7	8	9
No. of days :	9	43	64	62	42	36	22	14	6	2

Test at the 0.05 level of significance whether the daily number of power failures in this city is a random variable having the Poisson distribution with $\lambda = 3.2$.

15. (a) The following are the cholesterol contents, in milligrams per package, which four laboratories obtained for 6-ounce packages of three very similar diet foods :

	Diet Food		
	A	B	C
Lab			
1	3.4	2.6	2.8
2	3	2.7	3.1
3	3.3	3	3.4
4	3.5	3.1	3.7

Perform a two-way analysis of variance and test the null hypothesis concerning the diet foods and the laboratories at the 0.05 level of significance.

Or

(b) A farmer wishes to test the effects of four different fertilizers A, B, C, D on the yield of wheat. In order to eliminate sources of error due to validity in soil fertility, he uses the fertilizers in a Latin square arrangement as indicated below, where the number indicate yields in bushels per unit area. Perform an analysis of variance to determine if there is a significant difference between the fertilizers at 0.05 level of significance.

A18	C21	D25	B11
D22	B12	A15	C19
B15	A20	C23	D24
C22	D21	B10	A17