Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 13020

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

First Semester

Civil Engineering

MA 105 — MATHEMATICS – I

(Common to all branches)

(Regulation 2007)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Using the Cayley–Hamilton theorem, find the inverse of A where $A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$.

2. Write down the quadratic form corresponding to the following matrix.

$$A = \begin{pmatrix} 1 & 1 & -2 & 0 \\ 1 & -4 & 0 & 0 \\ -2 & 0 & 6 & -3 \\ 0 & 0 & -3 & 2 \end{pmatrix}$$

3. Find k so that the lines $\frac{x+1}{-3} = \frac{y+1}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y+5}{1} = \frac{z+6}{7}$ are perpendicular.

4. Find the equation of the sphere centered at (3, –2,2) and passing through the point (–1,3.4).

5. Determine the curvature of the cycloid $x = a(t - \sin t), y = a(1 - \cos 3t)$ at arbitrary point (x, y).

6. Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha - p = 0$, where α is a parameter.

7. If $f(x,y)=\frac{1}{x^2}+\frac{1}{xy}+\frac{(\log x-\log y)}{x^2+y^2}$, prove that $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+2f=0$.

8. Let $f(x,y)=x\sin y+e^x\cos y, x=t^2+1, y=t^2$ then find the value of $\left(\frac{\partial f}{\partial t}\right)_{t=0}$.

9. Solve $(D-1)(D^2-2D+2)y=e^x$

10. Solve $\frac{d^2y}{dx^2}-\frac{1}{x}\left(\frac{dy}{dx}\right)=\frac{12\log x}{x^2}$

PART B — (5 × 16 = 80 marks)

11. (a) Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to the sum of the squares and find the corresponding linear transformation.

Or

(b) Find the characteristic equation of the matrix $A=\begin{pmatrix}1&3&7\\4&2&3\\1&2&1\end{pmatrix}$. Show that the equation is satisfied by A and hence obtain the inverse of the given matrix using the Cayley-Hamilton theorem,

12. (a) Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Find also the equations and the points in which the S.D meets the given lines.

Or

(b) Find the equation of the right circular cone generated by the straight lines drawn from the origin to cut the circle through the three points (1,2,2), (2,1, –2) and (2, –2,1).

13. (a) Find the co-ordinates of the centre of curvature for any point (x,y) on the parabola $y^2=4ax$. Also, find the equation of the evolute of the parabola.

Or

(b) Find the envelope of the line $\frac{x}{a}+\frac{y}{b}=1$ where $\frac{a}{h}+\frac{b}{k}=1$.

14. (a) Find the minimum distance of the origin from the plane whose equation is given by $ax+by+cz+d=0$.

Or

(b) If $z = xy$ where $x = r\cos\theta$ and $y = r\sin\theta$, express z in terms of r and θ

Find $\left(\frac{\partial z}{\partial r}\right)$ and $\left(\frac{\partial z}{\partial \theta}\right)$.

15. (a) Solve $(D^4+2D^2+1)y = x^2 \cos x$.

Or

(b) Solve $x^3\left(\frac{d^3y}{dx^3}\right)+2x^2\left(\frac{d^2y}{dx^2}\right)+2y=10\left(x+\frac{1}{x}\right)$.

———————