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Question Paper Code : 33536

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Third Semester

Civil Engineering

MA 1201/070030007/070030005/070030004 — MATHEMATICS — III

(Common to all Branches)

(Regulation 2004/2007)

(Common to B.E.(Part-Time) Second Semester, Civil Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering and Mechanical Engineering, Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the function ϕ from $z = \phi(xy)$.

2. Find the general solution of the Lagrange linear equation given by $p\ yz + q\ zx = xy$.

3. Write down the form of the Fourier series of an odd function in $(-l, l)$ and associated Euler's formulas for Fourier coefficients.

4. State Parseval's theorem.

5. Write down the three mathematically possible solutions of one dimensional wave equation.

6. Write down the form of the general solution of one dimensional heat flow equation, when both ends of the rod are insulated.

7. Find the Fourier cosine transform of $2e^{-5x} + 5e^{-2x}$.

8. State and prove the modulation theorem.

9. Prove that $Z(f(t+T)) = [\bar{f}(z) - f(0)]$, where $\bar{f}(z) = Z[f(t)]$.

10. Write down the formula for finding $Z^{-1}(\bar{f}(z))$ using Cauchy's residue theorem

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation by eliminating arbitrary functions f and g from $z = f(x+iy) + (x+iy)g(x-iy)$. (6)

(ii) Solve : $(D^2 - 3DD' + 2D'^2)z = (2+4x)e^{x+2y}$. (10)

Or

(b) (i) Solve : $z = px + qy - 4p^2q^2$. (8)

(ii) Solve, by using Lagrange's multipliers

$(y^2 + z^2 - x^2)p - 2xy\,q + 2xz = 0$. (8)

12. (a) (i) Obtain the Fourier series of the periodic function defined by

$$f(x) = \begin{cases} -\pi & \text{if} \quad -\pi < x < 0 \\ x & \text{if} \quad 0 < x < \pi \end{cases}.$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$. (8)

(ii) Find the complex form of the Fourier series of the function

$f(x) = e^{-x}$ in $-1 < x < 1$. (8)

Or

(b) (i) Expand $f(x) = x - x^2$ as a Fourier series in $-1 < x < 1$ and also find the root mean square value of $f(x)$ in the interval. (8)

(ii) Find the Fourier series upto third harmonic for the following data : (8)

x :	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$
y :	0	9.2	11.4	17.8	17.3	11.7

13. (a) A string is stretched and fastened to two points $x=0$ and $x=l$ apart. The $\frac{l}{3}$ point of the string is displaced to a distance 'h' and is released from rest. Find the displacement at any point of the string at any time t. (16)

Or

(b) The ends A and B of a rod of 30 cms length have their temperature kept at 20°C and the other at 80°C, until steady conditions prevail. The temperature of the end B is suddenly reduced to 60°C and kept so while the end A is raised to 40°C. Find the temperature distribution in the rod after time t. (16)

14. (a) (i) Find the Fourier transform of $f(x)=\begin{cases} a-|x| & \text{for} \quad |x|<a \\ 0 & \text{for} \quad |x|>a>0 \end{cases}$ and hence prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$. (8)

(ii) Find the Fourier consine transform of $e^{-a^2x^2}$ and hence evaluate Fourier sine transform of $xe^{-a^2x^2}$. (8)

Or

(b) (i) State and prove convolution theorem of Fourier transforms. (8)

(ii) Find Fourier consine and sine transform of x^{n-1}. (8)

15. (a) (i) Find $Z\left[\frac{1}{(n+1)(n+2)}\right]$ and $Z^{-1}\left[\frac{z^2-3}{(z+1)(z^2+1)}\right]$. (8)

(ii) Solve $y_{n+2}-4y_{n+1}+4y_n=0$ where $y_0=1, y_1=0$. (8)

Or

(b) (i) Find $Z(e^{-iat})$ and hence deduce the values of $Z(\cos at)$ and $Z(\sin at)$. (8)

(ii) Use convolution theorem to find the inverse Z transform of $\frac{z^2}{(z+a)(z+b)}$. (8)
