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Question Paper Code : 31521

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve $(D^2 - 4)y = 1$.
2. Convert $(3x^2D^2 + 5xD + 7)y = 2/x \log x$ into an equation with constant coefficients.
3. Define solenoidal vector function. If $\vec{V} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + 2\lambda z)\vec{k}$ is solenoidal, find the value of λ .
4. State Green's theorem.
5. Find the constants a, b if $f(z) = x + 2ay + i(3x + by)$ is analytic.
6. Find the critical points of the transformation $w = 1 + \frac{2}{z}$.
7. Evaluate $\int_C \frac{z+4}{z^2+2z}$ where C is the circle $|z - \frac{1}{2}| = \frac{1}{3}$.
8. Find the residue of $f(z) = \frac{1-e^{-z}}{z^3}$ at $z = 0$.
9. Find the Laplace transform of $f(t) = \frac{1-e^{-t}}{t}$.
10. Find the Laplace transform of the function $f(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 8x e^x \sin x$. (8)
(ii) Solve by the method of variation of parameters (8)
 $2\frac{d^2y}{dx^2} + 8y = \tan 2x$.

Or

- (b) (i) Solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + y = e^{e^{\log x}}$. (8)
(ii) Solve $\frac{dx}{dt} + 4x + 3y = t$; $\frac{dy}{dt} + 2x + 5y = e^{2t}$. (8)

12. (a) (i) Show that the vector field $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is irrotational. Find its scalar potential. (6)
- (ii) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle formed by the lines $x = -a$, $x = +a$, $y = 0$ and $y = b$. (10)
- Or
- (b) (i) Find a and b so that the surfaces $ax^3 - by^2z - (a+3)x^2 = 0$ and $4x^2y - z^3 - 11 = 0$ cut orthogonally at the point $(2, -1, -3)$. (6)
- (ii) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$, where S is the surface of the cube formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (10)
13. (a) (i) Prove that $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the corresponding analytic function and the imaginary part. (8)
- (ii) Find the bilinear map which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also find the image of the unit circle of the z plane. (8)
- Or
- (b) (i) Prove that $w = \frac{z}{1-z}$ maps the upper half of the z -plane to the upper half of the w -plane and also find the image of the unit circle of the z plane. (8)
- (ii) Find the analytic function $f(z) = u + iv$ where $v = 3r^2 \sin 2\theta - 2r \sin \theta$. Verify that u is a harmonic function. (8)
14. (a) (i) Find the residues of $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ at its isolated singularities using Laurent's series expansion. (8)
- (ii) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$, using contour integration. (8)
- Or
- (b) (i) Show that $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx = \frac{5\pi}{2}$. (8)
- (ii) Evaluate $\int_C \frac{z+1}{(z^2 + 2z + 4)^2} dz$, where C is the circle $|z+1+i|=2$, by Cauchy's integral formula. (8)
15. (a) (i) Evaluate $L^{-1}\left(\frac{3s^2 + 16s + 26}{s(s^2 + 4s + 13)}\right)$. (8)
- (ii) Find the inverse Laplace transform of the following : $\log\left(\frac{s+1}{s-1}\right)$. (8)
- Or
- (b) (i) Find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$ and find $L^{-1}\left[\frac{1}{(s^2 + a^2)^2}\right]$ and hence find $L^{-1}\left(\frac{1}{(s^2 + 9s + 13)^2}\right)$. (3 + 3 + 2)
- (ii) Using Laplace transforms, solve $y'' + y' = t^2 + 2t$, $y(0) = 4$ and $y'(0) = -2$. (8)