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Question Paper Code : 31519

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to All Branches)

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue.
2. Discuss the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.
3. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 6x - 6y + 8z + 9 = 0$.
4. Prove that the equation $x^2 - 2y^2 + 3z^2 + 5yz - 6zx - 4xy + 8x - 19y - 2z - 20 = 0$ represents a cone with vertex $(1, -2, 3)$.
5. Find the radius of curvature of the curve $xy = c^2$ at (c, c) .
6. Find the envelope of the lines $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$, θ being the parameter.
7. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

8. Find the Taylor series expansion of x^y near the point $(1,1)$ upto the first degree terms.

9. Evaluate $\int_0^{\pi} \int_0^{\sin \theta} r dr d\theta$.

10. Change the order of integration in $\int_0^1 \int_0^{2\sqrt{x}} f(x, y) dy dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}. \quad (8)$$

(ii) Using Cayley-Hamilton theorem find A^{-1} for the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}. \quad (8)$$

Or

(b) Reduce the quadratic form $Q = 3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6xz$ to its canonical form using orthogonal transformation. Also find its rank, index and signature. (16)

12. (a) (i) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 + 2x - 2y + 4z - 19 = 0$ and $x + 2y + 2z + 7 = 0$. (8)

(ii) Find the equation of the cone whose vertex is the point $(1, 1, 0)$ and whose base in the curve $y = 0, x^2 + z^2 = 4$. (8)

Or

(b) (i) Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. (8)

(ii) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9, x + y + z = 3$. (8)

13. (a) (i) Prove that for the curve $y = \frac{ax}{a+x}$, $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. (8)

(ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. (8)

Or

(b) (i) Obtain the equation of the evolute of the curve $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$. (8)

(ii) Prove that the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a, 0)$ is $\frac{3a}{2}$. (8)

14. (a) (i) If $z = f(x, y)$ and $x = r \cos \theta$, $y = r \sin \theta$ prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2$. (8)

(ii) Discuss the maxima and minima of $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$. (8)

Or

(b) (i) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ prove that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4$. (6)

(ii) A rectangular box open at the top is to have a capacity of 108 cu.ms. Find the dimensions of the box requiring the least material for its construction. (10)

15. (a) (i) Evaluate $\iint xy dx dy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$. (6)

(ii) Find the value of $\iiint xyz dx dy dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$. (10)

Or

(b) (i) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dy dx$ and hence evaluate it. (8)

(ii) Evaluate, by changing to polar co-ordinates, the integral $\int_0^{4a} \int_{\frac{y^2}{4a}}^y \frac{x^2 - y^2}{x^2 + y^2} dx dy$. (8)