Reg. No. :

**Question Paper Code : 75467**

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Third Semester

Software Engineering

EMA 003 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the partial differential equation corresponding to $z = (x+a)(y+b)$ where $a$ and $b$ are arbitrary constants.

2. Find the complete integral of $p = e^q$.

3. State Dirichlet's conditions under which $f(x)$ can be represented by Fourier series.

4. State Parseval's identity for half range expansion of $f(x)$ as a Fourier cosine series in $(0, l)$.

5. State Fourier integral theorem.

6. Find the Fourier sine transform of $\frac{1}{x}$.

7. State the conditions for existence of Laplace transform.

8. Find $L^{-1}[1]$.

9. Find $Z[1]$.

10. State final value theorem on Z-transforms.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$. (5)

(ii) Solve: $p(1+q) = qz$. (5)

(iii) Solve: $(D^2 - 4DD' + 4D'^2)\, z = e^{2x+y}$. (6)

Or

(b) Solve the following:

(i) $z = px + qy + \sqrt{1 + p^2 + q^2}$. (5)

(ii) $(y - z)p + (z - x)\, q = x - y$. (5)

(iii) $(D^2 - 3DD' + 2D'^2)\, z = \sin x \cos y$. (6)

12. (a) (i) If $f(x) = \begin{cases} -k & when\ -\pi < x < 0 \\ k & when\ 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$ for all $x$, derive the Fourier series for $f(x)$. Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \ldots\ldots to\ \infty$. (8)

(ii) Find the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. Hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots to\ \infty$. (8)

Or

(b) (i) Find the Fourier series of periodicity $2\pi$ for $f(x) = \left[\begin{array}{ll} x & in\ (0, \pi) \\ 2\pi - x & in\ (\pi,\ 2\pi) \end{array}\right.$ and hence deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.\infty = \frac{\pi^2}{8}$. (10)

(ii) Obtain the half range sine series of $e^x$ in $0 < x < 1$. (6)

13. (a) (i) Find the Fourier transform of $f(x)=\begin{cases}1 & if\ |x|<a\\ 0 & if\ |x|>a>0\end{cases}$ and hence evaluate $\int_0^{\infty}\frac{\sin x}{x}\,dx$. (10)

(ii) Solve for $f(x)$ the following integral equation $\int f(x)\sin\lambda x\,dx=\begin{cases}1 & if\ 0\le\lambda<1\\ 2 & if\ 1\le\lambda<2\\ 0 & if\ \lambda\ge 2\end{cases}$. (6)

Or

(b) (i) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$ and hence find the value of $\int_0^{\infty}\frac{\sin^4 x}{x^4}\,dx$. (10)

(ii) Find the Fourier sine and cosine transform of $f(x)=e^{-ax}\ a>0$. (6)

14. (a) (i) Find the Laplace transform of the triangular wave function of period $2a$ given by $f(t)=\begin{cases}t & if\ 0<t<a\\ (2a-t) & if\ a<t<2a\end{cases}$. (8)

(ii) Using convolution theorem, find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$. (8)

Or

(b) (i) Find: (1) $L\left(te^{-2t}\sin 4t\right)$

(2) $L^{-1}\left(\cot^{-1}(as)\right)$. (6)

(ii) Solve: $\frac{d^2y}{dx^2}+9y=\cos 2x$, given $y(0)=1$, $y(\pi/2)=-1$. (10)

15. (a) (i) Find: (1) $Z\left[\frac{1}{n(n+1)}\right]$

(2) $Z^{-1}\left[\frac{z}{z^2+11z+24}\right]$. (6)

(ii) Solve the following finite difference equation: (10)

$y_{n+2}+6y_{n+1}+9y_n=2^n$, given $y_0=y_1=0$.

Or

(b) (i) Find the inverse Z transform of $\dfrac{z^2}{\left(z-\dfrac{1}{2}\right)\left(z-\dfrac{1}{4}\right)}$ using convolution theorem. (6)

(ii) Using Z-transform solve $y_{n+2} \quad 2y_{n+1} + y_n = 2^n$ with $y_0 = 2$ and $y_1 = 1$. (10)

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