Reg. No. :

Question Paper Code : 75531

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Second Semester

Software Engineering

XCS 122/10677 SW 202 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to 5 year M.Sc. Information Technology/M.Sc. Computer Technology)

(Regulation 2003/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the limits of integration in the double integral $\iint_R xy\, dx\, dy$, where R is the region bounded by the line $x + 2y = 2$, lying in the first quadrant.

2. Evaluate $\int_0^1 \int_0^3 \int_0^4 y\, dx\, dy\, dz$.

3. Find the normal to the surface $x^3 - xyz + z^3 = 1$ at the point (1,1,1).

4. Find the work done when a force $\overline{F} = (x^2 - y^2 + x)\bar{i} - (2xy + y)\bar{j}$ displaces a particle in the xy plane from (0,0) to (1,1) along the curve $y = x$.

5. If a line makes angles α, β, γ with the co-ordinate axes, prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

6. Show that the sphere whose centre is (1,2,–2) and radius is 3, passes through the origin.

7. Is the function $f(z) = \overline{Z}$ analytic? Why?

8. Prove that $u = \sinh x . \sin y$ is harmonic.

9. State Laurant's theorem.

10. Evaluate $\int_C \frac{Sinz}{\left(z - \pi/2\right)^2} dz$ where C is the circle $|Z| = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Change the order of integration in $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y\,dx\,dy$ and then evaluate it. (8)

(ii) Evaluate $\iiint_V \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$, where V is the region of space bounded by the co-ordinate planes and the sphere $x^2+y^2+z^2=1$ and contained in the positive octant. (8)

Or

(b) (i) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$ by double integration. (8)

(ii) Find the volume of the tetrahedron bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$. (8)

12. (a) (i) Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)\overline{i} + (3xz + 2xy)\overline{j} + (3xy - 2xz + 2z)\overline{k}$ is both solenoidal and irrotational. (8)

(ii) Verify Green's theorem in a plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$, Where C is the boundary of the region defined by the lines $x = 0, y = 0$ and $x + y = 1$. (8)

Or

(b) (i) Use Stoke's theorem to find the value of $\int_C \overline{F}.d\overline{r}$, when $\overline{F} = (xy - x^2)\overline{i} + x^2 y\,\overline{j}$ and C is the boundary of the triangle in the XOY plane formed by $x = 1, y = 0$ and $y = x$. (8)

(ii) Use divergence theorem to evaluate $\iint_S (yz^2\overline{i} + zx^2\overline{j} + 2z^2\overline{k})d\overline{S}$, where S is the closed surface bounded by the XOY plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane. (8)

13. (a) (i) Find the equation to the plane passing through the points (1,-1,1) and (2,1,0) and perpendicular to the plane $2x - y + 4z - 1 = 0$. (8)

(ii) Show that the lines $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z-3}{4}$ and $\frac{x-1}{4} = \frac{y-1}{3} = \frac{z-1}{5}$ are coplanar. Also find the equation to the plane containing them. (8)

Or

(b) (i) Find the equation to the tangent plane at (1,–1, 2) to the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z - 12 = 0$. (6)

(ii) Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Also find the equation to the line of shortest distance. (10)

14. (a) (i) Prove that an analytic function whose imaginary part is constant is itself a constant. (6)

(ii) Find the function $f(z) = u + iv$ such that $f(z)$ is analytic, given that $u - v = e^x(\cos y - \sin y)$. (10)

Or

(b) (i) Show that if u and v are conjugate harmonic functions, the product uv is a harmonic function. (6)

(ii) Find the constant 'a' so that $u(x, y) = \alpha x^2 - y^2 + xy$ is harmonic. Find an analytic function $f(z)$ for which u is the real part. Also find its harmonic conjugate. (10)

15. (a) (i) Find the Taylor's series to represent $\frac{z^2 - 1}{(z+2)(z+3)}$ in $|z| < 2$. (8)

(ii) Use residue theorem to evaluate $\int \frac{3z^2 + z - 1}{(z^2 - 1)(z - 3)} dz$ around the circle $|z| = 2$. (8)

Or

(b) (i) Use contour integration technique to find the value of $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$. (8)

(ii) Evaluate $\int_C \frac{e^2 dz}{(z+2)(z+1)^2} dz$ where C is $|z| = 3$, using Cauchy's integral formula. (8)

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