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Reg. No. :

Question Paper Code : 75466

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Second Semester

Software Engineering

EMA 002 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_{-1}^{2}\int_{x}^{x+2} dy\,dx$.

2. Evaluate $\int_{0}^{2}\int_{1}^{3}\int_{1}^{2} xy^2\, z\,dz\,dy\,dx$.

3. Find the value of a if $\overline{F} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal.

4. Show that $div\, curl\, \overline{F} = 0$.

5. Find the value of k if the length of the perpendicular from the point (2, –1, 3) to the plane $2x - 2y + z + k = 0$ is 5 units.

6. Find the volume of the sphere $x^2 + y^2 + z^2 - 4x - 4y + 6z + 8 = 0$.

7. Define an analytic function $f(z)$.

8. Prove that the real and imaginary parts of an analytic function $f(z)$ are harmonic functions.

9. State Cauchy's integral formula.

10. Define a double pole with an example.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Change the order of integration in $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\, dx\, dy$ and hence evaluate. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = 25$ by triple integrals. (8)

Or

(b) (i) Show by double integration that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$. (8)

(ii) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by triple integrals. (8)

12. (a) Verify Gauss divergence theorem for $\overline{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$; $y = 0$, $y = 1$; $z = 0$, $z = 1$. (16)

Or

(b) Verify Stroke's theorem for the vector field $\overline{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half of the surface $x^2 + y^2 + z^2 = 1$ bounded by its projection of the $x \circ y$ plane. (16)

13. (a) (i) Find the magnitude and equations to the line of the shortest distance between the lines $x - 10 = \frac{y-9}{3} = \frac{z+2}{-2}$ and $\frac{x+1}{2} = \frac{y-12}{4} = z - 5$. (8)

(ii) Find the equation of the sphere which has its centre at the point (–1,2,3) and touches the plane $2x - y + 2z = 6$. (8)

Or

(b) (i) Find the image of the line $x + 5 = \frac{y+7}{6} = z$ in the plane $2x - y + z + 3 = 0$. (8)

(ii) Obtain the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at the point (1, 2, – 2) and passes through the origin. (8)

14. (a) (i) Determine the analytic function whose imaginary part is $\left(\frac{x-y}{x^2+y^2}\right)$. (8)

(ii) Show that the function $u(x,y)=e^{-2xy}\sin(x^2-y^2)$ is harmonic and find its conjugate. (8)

Or

(b) (i) Verify whether the function $v(x,y)=\tan^{-1}\left(\frac{y}{x}\right)$ is harmonic and find its conjugate harmonic function. (8)

(ii) If $f(z)$ is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f(z)|=0$. (8)

15. (a) (i) State and prove Cauchy's integral theorem. (8)

(ii) Evaluate by Contour integration $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$. (8)

Or

(b) (i) Obtain the Laurent's series the function $f(z)=\frac{1}{(z-1)(z-2)}$ in the region $0<|z-1|<1$. (8)

(ii) Apply the calculus of residues, to prove $\int_0^{\infty}\frac{dx}{(1+x^2)^2}=\frac{\pi}{4}$. (8)
