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Question Paper Code : 75469

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fourth Semester

Software Engineering

EMA 005 — Discrete Mathematics

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $\neg(P \wedge Q) \to (\neg P(\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$.

2. Give the Symbolic form of the statement "Every one who is healthy person can do all kinds of work".

3. If relations R and S are both reflexive, show that It RUS is also reflexive.

4. Let $f : R \to R$ be given by $f(x) = x^3 - 2$. Find f^1.

5. Show that $(Z_{5,} +_5)$ is a cyclic group.

6. State Lagrange's theorem.

7. The ring of even integers is a subring of the ring of integers- Comment.

8. Show that in a field F

   (a) $(-a)b = -(ab)$

   (b) $(-a^{-1})^{-1} = a$.

9. In any Boolean algebra, Show that $a = b$ if and only if $a\bar{b} + \bar{a}b = 0$.

10. Define distributive lattice and prove that every chain is a distributive lattice.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that S is valid inferences from the premises (8)

$P \to \rceil Q, Q \vee R, \rceil S \to P$ and $\rceil R$.

(ii) Obtain PDNF and PCNF of the formula $(\rceil P \vee \rceil Q) \to (P \leftrightarrow \rceil Q)$. (8)

Or

(b) (i) Show that the premises $R \to \rceil Q, R \vee S, S \to \rceil Q, P \to Q, P$ are inconsistent. (8)

(ii) Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than peter. Therefore John is not successful than peter. (8)

12. (a) (i) If r_1 and r_2 are equivalence relation in a set A, then prove $r_1 \cap r_2$ is an equivalence relation in A. (8)

(ii) Show that $\sim \sim A = A$. (8)

Or

(b) (i) Let the relation R be defined on the set of all real numbers by 'if x, y are real numbers, $xRy \Leftrightarrow x - y$ is a rational number'. Show that R is an Equivalence relation. (8)

(ii) Show that there exists a one-to-one mapping from $A \times B$ to $B \times A$. Is it also onto. (8)

13. (a) (i) Let $\langle H, * \rangle$ be a sub group of $\langle G, * \rangle$. Then show that $\langle H, * \rangle$ is a normal subgroup iff $a * h * a^{-1} = H, \forall\, a \in G$. (8)

(ii) State and prove Lagrange's theorem. (8)

Or

(b) (i) Prove that the direct product of two groups is a group. (8)

(ii) Show that $(m, m+1)$ parity check code $e = B^m \to B^{m+1}$ is a group code. (8)

14. (a) If R is a ring such that $a^2 = a;\ a \in R$. Prove that

(i) $a + a = 0;\ a \in R$ i.e. each elements of R is its own additive inverse.

(ii) $a + b = 0 \rightarrow a = b$

(iii) R is a commutative ring. (16)

Or

(b) Let S be a set of all 2×2 matrices of the form $\begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix}$ where a,b are integers. Show that S is a ring 7not a field. (16)

15. (a) (i) Establish Demorgon's laws in Boolean algebra. (8)

(ii) Show that if L is distributive lattice then for all $a,b,c \in L, (a*b)\oplus(b*c)\oplus(c*a) = (a\oplus b)*(b\oplus c)*(c\oplus a)$. (8)

Or

(b) (i) In a lattice show that $a \le b$ and $c \le d$ implies $a*c \le b*d$. (8)

(ii) Show that in a Boolean algebra $\overline{(a*b)} = \bar{a} \oplus \bar{b}$ and $\overline{(a\oplus b)} = \bar{a}*\bar{b}$. (8)