Reg. No. : | | | | | | | | | | | |

Question Paper Code : 75526

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

First Semester

Software Engineering

XCS 112/10677 SW 102 — TRIGONOMETRY, ALGEBRA AND CALCULUS

(Common to 5 Year M.Sc. Information Technology/M.Sc. Computer Technology)

(Regulation 2003/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $x = \cos\theta + i\sin\theta$ then find the value of $x^2 + \frac{1}{x^2}$?

2. Separate real and imaginary part of $\sin h(x+iy)$.

3. Find the rank of $\begin{bmatrix} 1 & -1 & 1 & 4 \\ 2 & 1 & 1 & -3 \\ 5 & 1 & 3 & -2 \end{bmatrix}$.

4. Write the matrix of the auadratic form $2xy + 2yz + 2zx$.

5. If $u = x^3 y^2 + x^2 y^3$ where $x = at^2$ and $y = 2at$ find $\frac{du}{dt}$?

6. If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(u, v)}{\partial(r, \theta)}$.

7. Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$.

8. Write the formula for the length of the arc of the curve $y = f(x)$ between the points $x = a$ and $x = b$.

9. Find particular integral of $(D^2 + 4)y = \cos 2x$.

10. Solve $(x^2 D^2 - 3xD + 4)\,y = 0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that

$$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\frac{n\theta}{2}. \quad (8)$$

(ii) Find $\dfrac{\cos 7\theta}{\cos\theta}$ in powers of $\cos\theta$. (8)

Or

(b) (i) Prove that $\sin^5\theta = \dfrac{1}{16}[\sin 5\theta - 5\sin 3\theta + 10\sin\theta]$. (8)

(ii) If $\tan(A + iB) = x + iy$ prove that $x^2 + y^2 + 1 - 2y\cosh 2B = 0$. (8)

12. (a) (i) Find the values of λ and μ for which

$x + y + 2z = 3$

$2x - y + 3z = 4$

$5x - y + \lambda z = \mu$.

have

(1) An unique solution

(2) Many solutions

(3) No solution. (8)

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and hence find A^{-1}. (8)

Or

(b) (i) Find the eigen values and eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$. (8)

(ii) Diagonalise $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ whose eigen values are 5, 1, 1. (8)

13. (a) (i) Find the Taylor's series expansion of $e^x \sin y$ at $\left(-1, \frac{\pi}{4}\right)$ up to third degree terms. (8)

(ii) If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$ show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$. (8)

Or

(b) (i) Examine the function $f(x, y) = x^3y^2(12 - x - y)$ for extreme values. (8)

(ii) If $u = x - y$, $v = y - z$, $w = z - x$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (8)

14. (a) (i) Find a reduction formulae for $\int \sin^4 x\,dx$, n being positive integer. (8)

(ii) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

Or

(b) (i) Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$. (8)

(ii) Find the volume of the sphere of radius a. (8)

15. (a) (i) Solve $(D-2)^2 y = 8\left(e^{2x} + \sin 2x + x^2\right)$. (8)

(ii) Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameter. (8)

Or

(b) (i) Solve $\left(x^2D^2 + 8xD + 13\right)y = \log x$. (8)

(ii) Solve $\left(D^2 + 4D + 3\right)y = e^{-x}\sin x + 10$. (8)
