Reg. No. :

Question Paper Code : 75727

5 Year M.Sc. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Software Engineering

EMA 001 — TRIGNOMETRY, ALGEBRA AND CALCULUS

(Regulation 2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Express $\frac{2-\sqrt{3}i}{1+i}$ in the form of $a+ib$.

2. Prove that $\cosh^2 x - \sinh^2 x = 1$.

3. What do you mean by consistent and inconsistent system of equations.

4. When is a Quadratic form said to be singular? What is its rank then?

5. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ using Euler's theorem.

6. If $x = r\cos\theta, y = r\sin\theta$. Find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^9\theta\, d\theta$.

8. Find the area enclosed by the $x-axis$, the curve $y = 4+\cos x$ and the ordinates $x=0$ and $x=2\pi$.

9. Solve the DE $\left[D^2 - D + 1\right]y = 0$.

10. Find the particular integral of $(D-3)^2 y = xe^{-2x}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If α, β be the roots of $x^2 - 2x + 4 = 0$. Prove that $\alpha^2 + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$. (8)

(ii) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$. Hence evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$. (8)

Or

(b) (i) If $\cosh(u+iv) = x+iy$ prove that $\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$ and $\frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$. (8)

(ii) Find the value of $(-1)^{\frac{1}{6}}$. (8)

12. (a) (i) Investigate for what values of λ, μ the equation $x+y+z=6, x+2y+3z=10$ and $x+2y+\lambda z = \mu$ have (1) no solution (2) a unique solution and (3) an infinite number of solution. (8)

(ii) Verify Caylery-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

Hence find A^4. (8)

Or

(b) Reduce the Q.F $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_2$ to canonical form and hence find the rank 1 index signature and nature of the Q.F. (16)

13. (a) (i) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x\frac{\partial y}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. (8)

(ii) Expand $e^x \cos y$ impowers of x and y as for as the terms of the 3rd degree. (8)

Or

(b) (i) By using the transformation $u = x + y$ and $v = x - y$ change the independent variables x and y in the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ to u and v. (8)

(ii) Examine $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme value. (8)

14. (a) (i) If $u_n = \int_0^a x^n e^{-x} dx$ prove that $un - (n-a)u_{n-1} + a(n-1)u_{n-2} = 0$. (8)

(ii) Find the length of the are of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latasrectum. (8)

Or

(b) (i) Find the area enclosed between $y^2 = 4ax$ and $x^2 = 4ay$ using integral. (8)

(ii) One quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ revolues about the x axis. Find the volume of the solid generated. (8)

15. (a) (i) Solve the DE $(D^2 + 2D + 1)y = x^3 + \cos 2x$. (8)

(ii) Solve the DE $\left[x^2 D^2 + 3xD + 5\right]y = \cos(\log x)$. (8)

Or

(b) (i) Solve $(2x+3)^2 \frac{d^2 y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$. (8)

(ii) Solve the simultaneous DES. $Dx + y = \sin t, x + Dy = \cos t$ given that $x = 2$ and $y = 0$ at $t = 0$. (8)

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