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Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 75536

5 Year M.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Third Semester

Computer Technology

XCS 231/10677 SW 301 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to 5 year M.Sc. Software Engineering/M.Sc. Information Technology)

(Regulation 2003/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation from $z = ax^n + by^n$.

2. Obtain the complete integral of $z = px + qy - 5\sqrt{pq}$.

3. Find the Fourier constant a_n for $f(x) = |x|$ in $-1 \le x \le 1$.

4. State Parseval's theorem for a function expressed in Fourier series.

5. Show that $F(f(ax)) = \frac{1}{|a|} F\left(\frac{s}{a}\right)$ where $F(s)$ is the Fourier transform of $f(x)$.

6. State the convolution theorem of the Fourier transform.

7. Find $L\left[\frac{1-e^t}{t}\right]$.

8. Determine $L^{-1}\left[\frac{5}{(s+5)(s+2)}\right]$.

9. Find $z[n(n-1)]$.

10. State the initial value theorem in z-transform.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. (8)

(ii) Solve $\left(D^2 + 4DD' - 5D'^2\right)z = x + y^2$. (8)

Or

(b) (i) Solve $x^2p^2 + y^2q^2 = z^2$. (8)

(ii) Form the partial differential equation from $z = f(2x+3y) + g(2x+y)$. (8)

12. (a) (i) Find the half range cosine series for $f(x) = \begin{cases} x^2; & 0 < x < 1 \\ 2-x; & 1 < x < 2 \end{cases}$. (8)

(ii) Find for $f(x) = x^3$ in $-\pi < x < \pi$, the Fourier series of periodicity 2π. (8)

Or

(b) Find the Fourier series of periodicity 2π for $f(x) = \begin{cases} x & \text{in } (0, \pi) \\ 2\pi - x & \text{in } (\pi, 2\pi) \end{cases}$ and hence deduce the value of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$. (16)

13. (a) (i) Find the inverse Laplace transform of $\left[\frac{5s-2}{s^2(s-1)(s+2)}\right]$. (8)

(ii) Solve the differentian equation $\left(D^2 + D - 2\right)y = 3\cos 3t - 11\sin 3t$; $y(0) = 0$; $y'(0) = 6$ using Laplace transform. (8)

Or

(b) Using Laplace transform solve the simultaneous equations $Dx + Dy = 1$ and $D^2x - y = e^{-t}$ given $x = 3$, $Dx = -2$ and $y = 0$ at $t = 0$. (16)

14. (a) Find Fourier sine and cosine transforms of e^{-ax}; $a > 0$ and hence deduce the value of the integral $\int_0^\infty \frac{x \sin \beta x}{a^2 + x^2}\, dx$ and the Fourier sine transform at $\frac{x}{a^2 + x^2}$. (16)

Or

(b) Show that the Fourier transform of $f(x) = \begin{cases} a - |x| & ; \text{ for } |x| < a \\ 0 & ; \text{ for } |x| > a \end{cases}$ is $\frac{\sqrt{2}}{\pi}\left(\frac{1 - \cos as}{s^2}\right)$. Hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. (16)

15. (a) (i) Find the z-transform of (1) $\frac{1}{n}$ (2) $\sin n\theta$. (8)

(ii) Solve using z-transform $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = y_1 = 0$. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{z^2}{\left(x-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right]$. (8)

(ii) Solve using z-transform $x(n+2) - 3x(n+1) + 2x(n) = 0$ given $x(0) = 0$ and $x(1) = 1$. (8)