Reg. No. :

Question Paper Code : 81749

M.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

First Semester

Structural Engineering

MA 9212/MA 9321/MA 903/UMA 9103/10211 AM 101 — APPLIED MATHEMATICS

(Common to M.E. Soil Mechanics and Foundation Engineering and M.E. (P.T) Structural Engineering and Construction)

(Regulation 2009/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What is α^2 in $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$?

2. A flexible stretched string is constrained to move with zero slope at one end $x = 0$ while the other end $x = L$ is held against any movement. The time dependent motion of the string is subjected to the initial displacement given by $y(x,0) = y_0 \cos\frac{\pi x}{2L}$ and is released from this position with zero velocity. Write the boundary conditions to find the displacement.

3. If $U(\alpha, t)$ is the Fourier transform of $u(x,t)$ and if both u and $\frac{\partial u}{\partial x}$ vanishes as $x \to \pm\infty$, find the Fourier transform of $\frac{\partial^2 u}{\partial x^2}$.

4. State any two properties of harmonic function.

5. Write the Ostrogradsky equation for the functional

$$\iiint_D \left\{ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 \right\} dx\, dy\, dz .$$

6. Write the Euler's equation for the functional $\int_0^{\pi}(y'^2 - y^2 + 4y\cos x)dx, y(0)=0,$ $y(\pi)=0$.

7. If 4 is the dominant eigen value of $A=\begin{pmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}$ and if –5 is the dominant eigen value of $B = A - 4I$, then find the least eigen value and the remaining eigen value of A.

8. What is the formula to obtain the second highest eigen value in the deflation method?

9. Write two point Gauss Hermite Quadrature formula.

10. Evaluate $\int_{-1}^{1}\int_{-1}^{1}(x^2 y + y^3 x + 1)\,dx\,dy$ using Gaussian quadrature formula.

PART B — (5 × 16 = 80 marks)

11. (a) A string is fixed at two points $x=0$ and $x=l$. Motion is initiated by displacing the string in the form $u=\lambda\sin\left(\frac{\pi x}{l}\right)$ and released from rest at time $t=0$. Find the displacement of the string using Lapalce transform method.

Or

(b) Using Fourier cosine transform find the temperature $u(x,t)$ in a semi-infinite rod $0\le x\le\infty$. Determine $u(x,t)$ for the PDE $u_t = k u_{xx}$, $0<x<\infty, t>0$ given that the initial condition is $u(x,0)=0, 0\le x\le\infty$, boundary conditions are $u_x(0,t)=-u_0$, a constant when $x=0$ and $t>0$ and $u, \frac{\partial u}{\partial x}\to 0$ as $x\to\infty$.

12. (a) Using the method of integral transform, solve the following PDE in the semi infinite strip described by $u_{xx}+u_{yy}=0$, $0<x<\infty, 0<y<a$ and the boundary conditions $u(x,0)=f(x), u(x,a)=0$, $u(x,y)=0$, $0<y<a, 0<x<\infty$ and $\frac{\partial u}{\partial x}\to 0$ as $x\to\infty$.

Or

(b) Solve the Poisson equation $\nabla^2 u = -x e^{-x^2}, -\infty<x<\infty,$ $u(x,0)=0$ $u(x,y)=0$ as $x, y\to\infty$, u is finite as $y\to\infty$.

13. (a) (i) Prove that the shortest distance between any two points in a plane is a straight line. (8)

(ii) Find an approximate solution to the problem of the minimum of the functional $\int_0^1 (y'^2 + y^2)dx$, $y(0)=0$, $y(1)=1$ by Ritz method. (8)

Or

(b) (i) Find the extremals of the functional $\int_0^{\frac{\pi}{2}} (y''^2 - y^2 + x^2)dx$ that satisfies the conditions $y(0)=1$, $y'(0)=0$, $y\left(\frac{\pi}{2}\right)=0$ and $y'\left(\frac{\pi}{2}\right)=-1$. (8)

(ii) Find the extremals of the isoperimetric problem $\int_0^1 (y'^2 + x^2)dx$ given that $\int_0^1 y^2\, dx = 2$, $y(0)=0$, $y(1)=0$. (8)

14. (a) (i) Using Faddeev-Leverrier method, find the characteristic equation and the eigen value of the matrix $\begin{pmatrix} -1 & 0 & 0 \\ 1 & -2 & 3 \\ 0 & 2 & -3 \end{pmatrix}$. (8)

(ii) Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ using power method corrected to two decimal places by assuming the initial vector as $(0, 1, 0)^T$. (8)

Or

(b) Using deflation method find the eigen value and the eigen vectors of the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ with $(1, 0)^T$ as the initial vector. (16)

15. (a) (i) Use Gauss Hermite four point Quarature formula to evaluate $\int_{-1}^{1} e^{-x^2} x^3 \, dx$. (8)

(ii) Using Gaussian three point formula evaluate $\int_{0}^{\frac{\pi}{2}} \sin t \, dt$. (8)

Or

(b) Find the area of the quadrilateral formed by $P(-1, 2)$, $Q(7, -3)$, $R(6,8)$, $S(2, 4)$ using mapping function. (16)
