Reg. No. :

Question Paper Code : 81755

M.E./M.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

First Semester

Communication Systems

MA 9218/MA 909 — APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Common to M.E. Computer and Communication, M.E. Digital Signal Processing and M.Tech Information and Communication Technology)

(Regulation 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

2. Define Bessel's equation of order n.

3. Give some properties of generalized inverse.

4. Define Hermitian matrix.

5. If Var $(X) = 4$, find Var $(4X + 5)$, where X is random variable.

6. A coin is tossed until head appears. Find the expected number of tosses required.

7. Find the value of k if $f(x,y) = k(1-x)(1-y); 0 < x, y < 1$ to be a joint density function of (X,Y).

8. Give the properties of correlation coefficient.

9. How many number of servers are found in self service queueing model.

10. Write down the Kendell's notation for queueing model.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{n=\infty} t^n J_n(x)$ (8)

(ii) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\cos x - \frac{3}{x}\sin x\right]$. (8)

Or

(b) State and prove orthogonality relation of Bessel function. (16)

12. (a) Find the generalized inverse of $A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$ by least square method. (16)

Or

(b) Find the QR decomposition of $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. (16)

13. (a) (i) Find the MGF of binomial distribution and hence find its mean and variance. (8)

(ii) In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a random variable having Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $k = 3$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on a given day? (8)

Or

(b) (i) The distribution function of a random variable X is given by $F(X) = 1-(1+x)e^{-x}; x \geq 0$. Find the density function, mean and variance of X. (8)

(ii) A discrete random variable X has the probability function shown below (8)

x	0	1	2	3	4	5	6	7
$p(x)$	0	a	$2a$	$2a$	$3a$	a^2	$2a^2$	$7a^2+a$

(1) Find a

(2) $P(X<6), P(X \geq 6), P(0<x<4)$

(3) $P(X<6/X \geq 4)$

(4) Find the smallest value of λ such that $P(X \leq \lambda) > \frac{1}{2}$.

14. (a) (i) Let X and Y be two random variables having the joint probability y function $f(x,y)=k(x+2y)$ where x and y can assume only the integer values 0,1 and 2. Find the marginal and conditional distributions. (8)

(ii) Let (X,Y) be two dimensional random variables with joint probability density function $f(x,y)=\begin{cases} 2 & ;0<x<y<1 \\ 0 & ;otherwise \end{cases}$. Find the correlation coefficient between x and y. (8)

Or

(b) (i) If X and Y are independent RV's having density functions $f_1(x)=\begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & elsewhere \end{cases}$, $f_2(x)=\begin{cases} 3e^{-3y} & y \geq 0 \\ 0 & elsewhere \end{cases}$, find the pdf of $U=X+Y$. (8)

(ii) The JPMF of X and Y is given by

X \ Y	0	1	2
0	0.1	0.1	0.2
1	0.2	0	0.1
2	0.1	0.1	0.1

(1) Find the correlation coefficient between X and Y

(2) Obtain the regression lines of X and Y. (8)

15. (a) Customers arrive at a one-man barber shop according to a Poisson process with in a mean arrival of 12 minutes. Customers spend an average of 10mts in the barber's chair.

(i) Calculate the expected number. of customers in the barber shop and in the queue?

(ii) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait?

(iii) How much time can a customer expect to spend in the barbers shop?

(iv) Management will provide another chair hire another barber, when a customer waiting time in the shop exceeds 1.25 hours. How much the average rate of arrivals increase to warrant a 2^{nd} barber?

(v) What is the average time customer spends in the queue?

(vi) What is the probability that the waiting time in the system is more than 30 minutes?

(vii) Calculate the percentage of customers who have to wait prior to getting into the barber's chair?

(viii) What is the probability that more than 3 customers are in the system? (16)

Or

(b) If $\lambda = 3$ per hour $\mu = 6$ per hour in a (M/M/1): (4/FIFO) system, then find the following

(i) Probability of no customer in the system

(ii) Probability of n customers in the system, where $n = 0.1,2,3,4$

(iii) Expected number of customers in the system

(iv) Expected number of customers in the queue

(v) The expected time a customer spends in the system

(vi) The expected waiting time of a customer in the queue. (16)