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Question Paper Code : 81750

M.E. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

First Semester

CAD/CAM

MA 9213/MA 905/10277 PS 101 — PROBABILITY AND STATISTICAL METHODS

(And also common for MA 9317 – Probability and Statistics for M.E. Industrial Engineering and M.E. Industrial Safety Engineering and M.E. CAD/CAM)

(Regulation 2009/2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X has the joint probability density function $f(x)=\begin{cases} k(x+1), 2 \le x \le 5 \\ 0, \quad \text{elsewhere.} \end{cases}$ Find $P(x<4)$.

2. Find the moment generating function of a random variable X if it has the joint probability density function $f(x)=\begin{cases} 2e^{-2x}, x \ge 0 \\ 0. \quad x<0 \end{cases}$.

3. If $r_{12}=0.80, r_{13}=-0.40$ and $r_{23}=-0.56$, Find $r_{12.3}$.

4. Find the maximum likelihood estimator for the parameter λ of a Poisson distribution on the basis of sample size n.

5. Define chi-square test for significance.

6. State any two assumptions of Analysis of variance.

7. Describe Latin Square design.

8. What is the main advantage of Latin Square design and Randomised Block design?

9. State any two merits of method of moving averages.

10. What are normal equations to fit the exponential curve $y=ab^x$?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) If the density function of a continuous random variable X is given by $f(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty$, Find the moment generating function of X. Also, find its mean and variance. (8)

(ii) Suppose the joint probability density function is given by

$$f(x,y) = \begin{cases} \frac{6}{5}(x + y^2), 0 \le x \le 1, 0 \le y \le 1, \\ 0, \quad \text{elsewhere.} \end{cases}$$

Obtain the marginal probability density function of X and that of Y. Hence or otherwise find $P\left[\frac{1}{4} \le y \le \frac{3}{4}\right]$. (8)

Or

(b) If the joint density function of the random variables X and Y is given by $f(x,y) = \begin{cases} (2 - x - y), 0 \le x \le 1, 0 \le y \le 1, \\ 0, \quad \text{elsewhere.} \end{cases}$ Find the correlation coefficient of X and Y. (16)

12. (a) (i) For the table of data given below:

X	25	28	35	32	31	36	29	38	34	32
Y	43	46	49	41	36	32	31	30	33	39

Find :

(1) The two regression equations

(2) The coefficient of correlation between marks in X and Y

(3) The most likely marks in Y when the marks in X are 30. (10)

(ii) In a trivariate distribution, $\sigma_1 = 3, \sigma_2 = \sigma_3 = 5$ and $r_{12} = 0.6$, $r_{23} = r_{31} = 0.8$. Find (A) $r_{23.1}$ (B) $R_{1.23}$. (6)

Or

(b) (i) For random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimators for (i) μ when σ^2 is known (ii) σ^2 when μ is known (iii) the simultaneous estimation μ and σ^2. (8)

(ii) A random variable X takes the values 0, 1, 2 with respective probabilities $\frac{\theta}{4N} + \frac{1}{2}\left(1 - \frac{\theta}{N}\right), \frac{\theta}{2N} + \frac{\alpha}{2}\left(1 - \frac{\theta}{N}\right)$ and $\frac{\theta}{4N} + \frac{1-\alpha}{2}\left(1 - \frac{\theta}{N}\right)$ where N is a known number α, θ are unknown parameters. If 75 independent observations on X yielded the values 0, 1, 2 with frequencies 27, 38, 10 respectively, estimate θ and α by method of moments. (8)

13. (a) (i) The life time of electric bulbs for a random sample of 10 from a large consignment gave the following data: (8)

Item:	1	2	3	4	5	6	7	8	9	10
Life in 1000 hours:	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

Test whether the average life time of electric bulbs can be 4000 hars at 5% level of significance.

(ii) The number of parts for a particular spare part in a factory was found to vary from day to day. In a sample study the following information was obtained:

Days:	Mon	Tue	Wed	Thu	Fri	Sat
No. of parts demanded:	1124	1125	1110	1120	1126	1115

Use Chi square test to test the number of parts demanded does not depend on the day of the week. (8)

Or

(b) To assess the significance of possible variation in performance in a certain test between the grammar schools of a city, a common test was given to a number of students taken at random from Fifth standard of each of the four schools concerned. The results are given below. Make an analysis of variance of data. (16)

Schools

A	B	C	D
8	12	18	13
10	11	12	9
12	9	16	12
8	14	6	16
7	4	8	15

14. (a) Three varieties of a crop are tested in a randomised block design with four replications, the layout being as given below: The yields are given in kilograms.

C48	A41	B52	A49
A47	B49	C52	C51
B49	C53	A49	B50

Perform the analysis of variance. (16)

Or

(b) Analyse the variance in the following Latin square of yields (in kgs) of paddy where A, B, C, D denote the different methods of cultivation.

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields. (16)

15. (a) (i) Compute the trend values for the following data using method of least squares. (8)

Year: X	2000	2001	2002	2003	2004	2005	2006
Y	83	60	54	21	22	13	23

(ii) The production of cement by a firm in years 1 to 9 is given below :

Year X	1	2	3	4	5	6	7	8	9
Production (tonnes) Y	4	5	5	6	7	8	9	8	9

Calculate the trend values for the above series by the following methods:

(1) 3 yearly moving average

(2) Linear trend. (8)

Or

(b) (i) The sales of a company for the last eight years are given below :

Year X	2000	2001	2002	2003	2004	2005	2006	2007
Sales Y (Rs. 1000s)	52	45	98	92	110	185	175	220

Estimate sale figure for 2008 using an equation of the form where X = years and Y = sales. (8)

(ii) Calculate the seasonal index from the following data using the average method: (8)

Year	I Quarter	II Quarter	III Quarter	IV Quarter
1992	72	68	80	70
1993	76	70	82	74
1994	74	66	84	80
1995	76	74	84	78
1996	78	74	86	82