Reg. No. : | | | | | | | | | | |

Question Paper Code: 45021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2018

Fifth Semester

Computer Science and Engineering

14UMA521 - DISCRETE MATHEMATICS

(Regulation 2014)

(Common to IT Branch)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. Which of the following statement is the negation of the statement "The crop will be destroyed if there is a flood" ?
   (a) The crop will not be destroyed, if there is a flood
   (b) The crop will not be destroyed, if there is no flood
   (c) Crops are destroyed during the flood
   (d) There is a flood and the crops are not destroyed

2. $P \rightarrow Q$is equivalent to
   (a) $\urcorner Q \rightarrow P$ (b) $Q \rightarrow P$ (c) $P \rightarrow \urcorner Q$ (d) $\urcorner Q \rightarrow \urcorner P$

3. The solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$ is
   (a) $a_n = 2^n + (-1)^n$ (b) $a_n = (1+n)(3)^n$
   (c) $a_n = 3.2^n - (-1)^n$ (d) $\left(1+\frac{n}{3}\right)(3)^n$

4. The numbers of ways in which 6 boys and 4 girls be arranged in a straight line so that no two girls are together is
   (a) 10^{P_6} (b) 604800 (c) 720 (d) 17280

5. A vertex of degree one is called
   (a) Isolated vertex (b) Unit vertex (c) Pendant vertex (d) Proper vertex

6. The number of vertices in a regular graph of degree 4 with 10 edges is
   (a) 4 (b) 10 (c) 6 (d) 5

7. The set of all real number usual multiplication is not a group, since
   (a) Multiplication is not a binary operation (b) Multiplication is not associative
   (c) Identity element does not exist (d) Zero has no inverse

8. The necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup when $a, b \in H$is
   (a) $a^{-1} * h * a \in H$ (b) $a^{-1} * b \in H$
   (c) $a^{-1} * b^{-1} \in H$ (d) $(a * b)^{-1} \in H$

9. A self-complimented, distributive lattice is called
   (a) Modular Lattice (b) Boolean Algebra
   (c) Complete Lattice (d) Self-Dual Lattice

10. Let $D_{30} = \{1, 2, 3, 4, 5, 6, 10, 15, 30\}$ and relation / (divides) be partial ordering on D_{30}. Then the least upper bound (lub) of 10 and 15 is
   (a) 30 (b) 15 (c) 10 (d) 6

PART - B (5 x 2 = 10 Marks)

11. Define quantifiers. What are its types.

12. Find the recurrence relation from $y_k = A2^k + B3^k$.

13. State any two properties of trees.

14. Draw all the spanning trees of $\boldsymbol{K_3}$.

15. Is the poset$(Z^+, /)$ a lattice?

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Obtain the principal disjunctive and principal conjunctive normal forms of $\big(P \to (Q \wedge R)\big) \wedge \big(\sim P \to (\sim Q \wedge \sim R)\big)$. (8)

(ii) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (8)

Or

(b) (i) Check whether the following set of premises are not valid: Whenever the system software is being upgraded, users cannot access the file system. If users can access the file systems, then they can save new files. If users cannot save new files, then the system software is not being upgraded. (8)

(ii) Show that $(\exists x)\big(P(x) \to Q(x)\big)$ follows from the premises $\exists x\big(P(x) \wedge Q(x)\big) \to (y)\big(R(y) \to S(y)\big)$ *and* $\exists y(R(y) \wedge \neg S(y))$. (8)

17.(a) (i) Solve the recurrence relation $y_{n+2} - 6y_{n+1} + 9y_n = 0,\ y_1 = 4$ *and* $y_0 = 1$. (8)

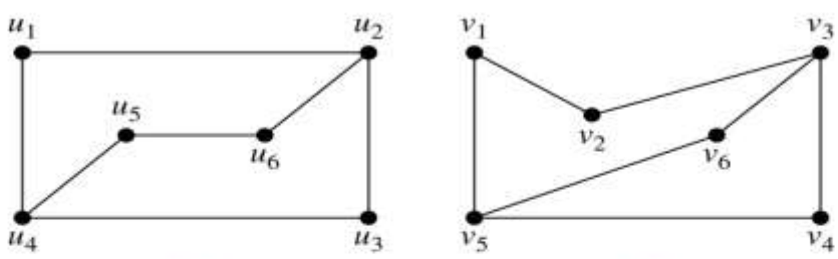
(ii) Find the number of integers between 1 and 100, both inclusive, that are divisible by 2, 3, 5, but not by 7. (8)

Or

(b) (i) Show that by mathematical induction principle, $3^{2n} + 4^{n+1}$ is divisible by 5, for $n \geq 0$. (8)

(ii) Find the number of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 and 7. (8)

18. (a) (i) Examine whether the following graphs are isomorphic. (8)



G H

(ii) Prove that a connected graph is Eulerian if all vertices are of even degree. (8)

Or

(b) (i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (8)

(ii) Find the adjacency matrix of the following graph G. Find A^2, A^3 and $Y = A + A^2 + A^3 + A^4$. What is your observation of entries in A^2 and A^3 ? (8)

19. (a) (i) State and prove Lagrange's theorem. (8)

(ii) Define subgroup with an example. Also prove that the intersection of two subgroups of a group is also a subgroup of the group. (8)

Or

(b) (i) Prove that a non-empty subset H of a group G is a subgroup if $a, b \epsilon H \Rightarrow a * b^{-1} \epsilon H$. (8)

(ii) Let $f: G \to G$ be a homomorphism of group G with kernel *K*. Then prove that *K* is a normal subgroup of *G* and *G/K* is isomorphic to image of f. (8)

20. (a) (i) State and prove DeMargon's law of lattice. (8)

(ii) In any Boolean algebra, show that $a'b + ab' = 0 \; iff \; a = b$. (8)

Or

(b) (i) State and prove distributive inequality of Lattice. (8)

(ii) In a complemented, distributive lattice, prove the following:
(i) $(ab)' + (a+b)' = a' + b'$ (ii) $ab'c + ab'c = b'c$ (8)