Reg. No. : | | | | | | | | | | |

Question Paper Code: 43021

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2018

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. The formula for finding the Euler constant a_n of a Fourier series in *[0, 2π]* is ________

   (a) $a_n = \int_0^{\pi} f(x) cosnx\, dx$ (b) $a_n = \int_0^{2\pi} f(x) cosnx\, dx$

   (c) $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) cosnx\, dx$ (d) $a_n = \frac{1}{\pi}\int_0^{\pi} f(x) cos\frac{n\pi x}{l}\, dx$

2. R.M.S value of f(x) = x in (-1,1) is

   (a) 0 (b) 1 (c) $\frac{1}{3}$ (d) $\sqrt{\frac{1}{3}}$

3. Find the Fourier sine transform of e^{-3x}

   (a) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+9}$ (b) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+9}$ (c) $\frac{a}{s^2+9}$ (d) $\frac{s}{s^2+9}$

4. Fourier sine transform of xf(x) is,

   (a) $F_c'(s)$ (b) $F_s'(s)$ (c) - $F_c'(s)$ (d) - $F_s'(s)$

5. $\lim_{z \to 1}$ (z-1) F(z) =

   (a) f(1) (b) $F(\infty)$ (c) $f(\infty)$ (d) f(0)

6. $Z\{cosn\theta\}$ is ________

(a) $\frac{(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (b) $\frac{Z\,(Z-cos\theta)}{Z^2-2Z\,cos\theta+1}$ (c) $\frac{Z}{Z^2-2Z\,cos\theta+1}$ (d) $\frac{1}{Z^2-2Z\,cos\theta+1}$

7. In one dimensional heat equation $u_t = \alpha^2 u_{xx}$. What is α^2?

(a) Velocity (b) Speed (c) Diffusivity (d) Displacement

8. A rod of length 40 cm whose one end is kept at 20°C and the other end is kept at 60°C is maintained so until steady state prevails. Find the steady state temperature at a location 15cm from A?

(a) 5 (b) 10 (c) 13 (d) 15

9. The finite difference approximation to $y_i' =$

(a) $\frac{y_{i+1}-y_{i-1}}{h}$ (b) $\frac{y_{i+1}+y_{i-1}}{h}$

(c) $\frac{y_{i+1}-y_{i-1}}{2h}$ (d) $\frac{y_{i+1}+y_{i-1}}{2h}$

10. Liebmann's iteration process is used to solve the _________

(a) One dimensional heat flow equation
(b) Two dimensional heat flow under steady state equation
(c) One dimensional wave equation
(d) Hyperbolic equation

PART - B (5 x 2 = 10 Marks)

11. Find the Fourier constants b_n for $x\,sinx$ in $(-\pi, \pi)$.

12. Find the Fourier sine transform of $\frac{1}{x}$.

13. State initial and final value theorem of Z – transform.

14. Define steady state condition on heat flow.

15. Write down the standard five-point formula to solve the Laplace equation.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Express $f(x) = (\pi - x)^2$ as a Fourier series of periodicity 2π in $0 < x < 2\pi$. (8)

(ii) Find the Fourier series for $f(x) = x^2$ in $-\pi, x < \pi$. (8)

Or

(b) (i) Find the cosine series for f (x) = x in (0, π) and then using Parseval's theorem, show that $\frac{1}{1^4}+\frac{1}{3^4}+....=\frac{\pi^4}{96}$. (8)

(ii) Find the complex form of Fourier series of $f(x)$ if $f(x)=\sin ax \; in \; -\pi<x<\pi$. (8)

17. (a) (i) Find the Fourier transform of $f(x)=\begin{cases}1, & |x|\le a\\ 0, & |x|>a\end{cases}$. Hence evaluate $\int_0^{\infty}\frac{\sin s}{s}ds$ and using Parseval's identity prove that $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$ (8)

(ii) Find the Fourier transform of $e^{-a^2x^2}$ Hence prove that $e^{\frac{-x^2}{2}}$ is self reciprocal with respect to Fourier transforms. (8)

Or

(b) (i) Show that Fourier Transform of $f(x) = e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. (8)

(ii) Evaluate $\int_0^{\infty}\frac{dx}{(4+x^2)\ (25+x^2)}$ using Fourier transform method. (8)

18. (a) (i) Find $Z(cosn\theta)$ and hence deduce $Z\left(\frac{cosn\pi}{2}\right)$ (8)

(ii) Using convolution theorem, find the inverse Z- transforms of $\frac{z^2}{(z+a)^2}$. (8)

Or

(b) (i) Solve $y_{n+2}+4y_{n+1}+3y_n=3^n$ with $y_0=0$and$y_1=1$. (8)

(ii) Find the inverse Z – transform of $\frac{z(z+1)}{(z-1)^3}$ by residue method. (8)

19. (a) A tightly stretched flexible string has its ends fixed at x = 0 and x = l. At time t = 0, the string is given a shape defined by f(x) = $k(lx-x^2)$where 'k' is a constant, and then released from rest. Find the displacement of any point x of the string at any time t > 0. (16)

Or

(b) A metal bar 20cm long, with insulated sides, has its ends A and B kept at 30ºC and 90ºC respectively until steady state conditions prevail. The temperature at each end is suddenly raised to 0ºC and kept so. Find the subsequent temperature at any time of the bar at any time. (16)

20. (a) Solve the Poisson's equations $\nabla^2 u = -81xy$, $0 < x < 1$, $0 < y < 1$, h=1/3, $u(0,y) = u(x,0)$, $u(1,y) = u(x,1) = 100$. (16)

Or

(b) (i) Solve u_{xx} =32 u_t with h=0.25 for t>0; 0<x<1 and u(x,0)=u(0,t)=0; u(1,t)=t .Tabulate u upto t=5 sec using Bender-Schmidt formula. (8)

(ii) Find the solution to the wave equations $u_{xx} = u_{tt}$,0 < x < 1, t > 0, given that $u_t(x,0)=0$, $u(1,t) = u(0,t) = 0$ and $u(x,0) = 100 \sin \pi x$. Compute u for 4 time steps with h=0.25. (8)
