Reg. No. : | | | | | | | | | | |

Question Paper Code: 33001

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2018

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Find the constant term in the Fourier series corresponding to $f(x) = \sqrt{1 - \cos x}$ expressed in the interval $(-\pi, \pi)$.

2. State the conditions for $f(x)$ to have Fourier series expansion.

3. Find the Fourier cosine transform of e^{-2x}.

4. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$.

5. Find the Z-transform of a^n.

6. Write the formula for $Z^{-1}[F(z)]$ using Cauchy's residue theorem.

7. State initial and final value theorems on z - transform.

8. What does a^2 represent in the equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$?

9. Write down the diagonal five point formula in Laplace equation.

10. State the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Fourier series expansion of $f(x) = x^2 + x \ in \ (-2, 2)$. Hence find the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \infty$ (8)

(ii) Find the Fourier series expansion of the function $f(x) = \begin{Bmatrix} 0 & : -\pi \leq x \leq 0 \\ sinx & : 0 \leq x \leq \pi \end{Bmatrix}$ (8)

Or

(b) (i) Find the Half range cosine series for $y = x$ in *(0, l)* and hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots \infty$. (8)

(ii) Compute the first two harmonics of the Fourier series of *f(x)* given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

12. (a) Find the Fourier cosine and sine transform of $e^{-ax}, a > 0$ and hence evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$ and $\int_0^\infty \frac{x^2}{(a^2 + x^2)^2} dx$ (16)

Or

(b) (i) Find the Fourier transform of $f(x) = \begin{Bmatrix} 1 - |x| & : |x| < 1 \\ 0 & : otherwise \end{Bmatrix}$ and hence find the value of $\int_0^\infty \frac{sin^4 t}{t^4} dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x\, e^{-x^2}$. (8)

13. (a) Find the inverse z - transform of $\frac{z^2}{(z-a)(z-b)}$ using convolution theorem. (16)

Or

(b) (i) State and prove initial and final value theorem on Z- transform. (8)

(ii) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$ by using method of Partial fraction. (8)

14. (a) The ends A and B of a rod l cm long have the temperature at 30^0c and 80^0c until steady state prevails. The temperature of the ends is then changed to 40^0c and 60^0c respectively. Find the temperature distribution in the rod at any time. (16)

Or

(b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = 3(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at any time t. (16)

15. (a) Solve numerically $4u_{xx} = u_{tt}$ with the boundary conditions $u(0,t) = 0,\quad u(4,t) = 0$ and the initial conditions $u_t(x,0) = 0$ and $u(x,0) = x\quad(4 - x)$ taking $h = 1$ up to *4* time steps. (16)

Or

(b) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0,\ x = 3,\ y = 3\ with\ u = 0$ on the boundary and mesh length 1 unit. (16)