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Reg. No. : | | | | | | | | | | |

Question Paper Code: 54024

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2018

Fourth Semester

Electronics and Communication Engineering

15UMA424 - PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2015)

(Statistical tables may be permitted)

Duration: Three hours   Maximum: 100 Marks

PART A - (10 x 1 = 10 Marks)

1. If X is the discrete random variable having the probability density function, then find k. CO1-App

x	-2	0	1
P(X)	8k	2k	3k

(a) 1/13   (b) -1/13   (c) 1   (d) -1

2. Six coins are tossed 6,400 times. What is the probability of getting 6 holds, *x* times? CO1-R

(a) e^{-100x} (b) $\frac{1}{x}\ e^{-100x}$ (c) $\frac{a-b}{\frac{1}{x!}e^{-100}\ 100^{x}}$ (d) None of these

3. $Cov(x+a, y+b) =$ CO2-App

(a) $abCov(x,y)$ (b) $(a+b)Cov(x,y)$ (c) $Cov(x,y)$ (d) 0

4. The regression coefficient b_{XY} = 0.2337 and b_{YX} = 0.6643 , then the correlation coefficient is CO2-R

(a) 0.498   (b) 0.394   (c) 0.239   (d) 0.25

5. Differences of two independent Poisson processes is CO3-R

(a) not Poisson process   (b) a Gaussian process

(c) a Binomial process   (d) a sine wave process

6. A random process $\{X(t)\}$ is said to be a first order stationary process if $\mu =$ CO3-R

(a) constant (b) function of t

(c) function of τ (d) none

7. In auto correlation function_____ CO4-R

(a) $R(\tau)$ is even (b) $R(\tau)$ is odd (c) $R(\tau)$ is both even and odd (d) None of these

8. The spectral density function of a real random process is CO4-R

(a) an odd function (b) neither odd nor even function

(c) an even function (d) straight line function

9. The convolution form of the output Y(t) of a linear time invariant system with the input X(t) and the system weighting function h(t) CO5-R

(a) $\int_{-\infty}^{\infty} h(u)\ du$ (b) $\int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$

(c) $\int_{-\infty}^{\infty} h(u)\, y(t-u)\, du$ (d) $\int_{-\infty}^{\infty} X(t-u)\, du$

10. If $S_{XX}(w) = \frac{N_0}{2}$ and $H(w) = 1$ then $S_{YY}(w) =$ CO5-R

(a) N_0 (b) $\frac{N_0}{2}$ (c) $\frac{2}{N_0}$ (d) None of the above

PART – B (5 x 2= 10Marks)

11. State Baye's theorem CO1-R

12. If the joint probability density function of (X,Y) is given by $f(x,y) = e^{-(x+y)}, x \geq 0, y \geq 0$ find E (XY). CO2-E

13. State Chapman-Kolmogorov theorem. CO3-R

14. Find Mean of the random process X(t) , where $R(\tau) = 16 + \frac{9}{1+16\tau^2}$ CO4-E

15. Describe a linear system. CO5-R

PART – C (5 x 16= 80Marks)

16. (a) (i) A continuous random variable X has a PDF $f(x) = 3x^2, 0 \leq x \leq 1$. Find the value of K and α such that (1) $P(X \leq K) = P(X > K)$ (2) $P(X > \propto) = 0.1$ CO1-App (8)

		(ii) The distribution function of the random variable X is given by $F(x) = 1- (1 + x)\, e^{-x}$, $x \geq 0$. Find the density function, mean and variance of X.	CO1-App	(8)

Or

	(b)	(i) Derive the moment generating function of Poisson distribution and hence obtain its mean and variance.	CO1-App	(8)
		(ii) State and prove memory less property of exponential distribution.	CO1-App	(8)
17.	(a)	(i) The joint pdf of random variable *(X,Y)* is given by $f(x,y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of k and also prove that *X* and *Y* are independent.	CO2-App	(8)
		(ii) The joint probability density function of random variable X and Y is given by $f(x,y) = 4xye^{-(x^2+y^2)},\ \ x > 0\ \ , y > 0.$ Check whether X and Y are independent or not..	CO2-App	(8)

Or

	(b)	(i) The joint probability mass function of *(X,Y)* is given by $p(x,y) = k(2x+3y),\ x = 0,1,2;\ y = 1,2,3.$ Find the value of k and all the marginal probability distribution function. Also find the probability distribution of *(X + Y).*	CO2-Ana	(8)
		(ii) The regression lines for two random variables X and Y are 8X – 10Y + 66 = 0 and 40X – 18Y = 214. Find (a) the mean values of X and Y (b) the correlation coefficient between X and Y.	CO2-Ana	(8)
18.	(a)	(i) Consider the random process $X(t) = Y\cos \omega t$, $t \geq 0$, Where ω is a constant and Y is a uniform random variable over (0,1). Find the auto correlation function $R(t, \tau)$ of X(t) .	CO3-Ana	(8)

(ii) The probability distribution of the process {X(t)} is given by CO3-Ana (8)

$$P[\,X(t) = n\,] = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, 3, \ldots\ldots\ldots.. \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

Show that it is not stationary.

Or

(b) Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as equally likely to throw the ball to B or to A. Show that the process is Markovian. Find the transition matrix and classify the states. CO3-Ana (16)

19. (a) State and Prove the Wiener–Khinchine theorem CO4-App (16)

Or

(b) (i) The power spectral density of the random process is given by CO4-App (8)

$$S_{XX}(\omega) = \begin{cases} S_0 & -a < \omega < a \\ 0 & otherwise \end{cases}.$$

Find the autocorrelation function and also the mean square value.

(ii) The cross-power spectrum of real random processes {X(t)} and {Yt)} is given by CO4-App (8)

$$S_{XY}(\omega) = \begin{cases} a + bj\omega & -1 < \omega < 1 \\ 0 & otherwise \end{cases}$$

Find the cross-correlation function .

20. (a) (i) A random process $\{X(t)\}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t},\ t > 0$. If the autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $Y(t)$. CO5-App (8)

(ii) If $\{X(t)\}$ is a WSS process with auto correlation function $R_{XX}(\tau)$, then prove that the power spectral density of the output $\{Y(t)\}$ is $S_{YY}(w) = S_{XX}(w)|H(w)|^2$ CO5-App (8)

Or

(b) A Linear system is described by the impulse response $h(t) = \beta e^{-\beta t}u(t)$. Assume an input process whose auto correlation function is $B\delta(\tau)$. Find the mean and Auto Correlation function of the output process CO5-App (16)