Reg. No. : | | | | | | | | | | |

Question Paper Code: 34024

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2018

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

(Statistical tables may be permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. Find c, if a continuous random variable X has the density function $f(x) = \dfrac{c}{1+x^2}, -\infty \leq x \leq \infty$.
2. Two dice are thrown 720 times; find the average number of times in which the number on the first dice exceeds the number on the second dice.
3. State the equations of the two regression lines. What is the angle between them?
4. If $Y = -2X + 3$, find the $Cov(X,Y)$.
5. Prove that a first order stationary random process has a constant mean.
6. State any two properties of Poisson process.
7. State any two properties of cross correlation function.
8. State any two properties of an auto correlation function.
9. Define White noise.
10. Define average power in the response of a linear system.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$3k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) the value of k, (ii) $P(1.5 < X < 4.5\,/\,X > 2)$ and (iii) the smallest value of λ for which $P(X \le \lambda) > \frac{1}{2}$. (8)

(ii) Show that sum of any two Poisson variables is again a Poisson variate. (8)

Or

(b) (i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act independently.

(1) What is the probability that all four lines are busy?

(2) What is the probability that atleast two of them are busy? (8)

(ii) Describe Gamma distribution, Obtain its moment generating function. Hence compute its mean and variance. (8)

12. (a) The joint probability density function of a random variable is given by

$$f(x,y)=\begin{cases} Kxye^{-(x^2+y^2)} & ,x>0, y>0 \\ 0\text{ , otherwise} \end{cases}$$

. Find the value of K and prove also that x and y are independent. (16)

Or

(b) (i) The joint pdf of X and Y is given by $f(x,y) = e^{-(x+y)}, x > 0, y > 0$. Find the probability density function of $U = \frac{X+Y}{2}$. (16)

13. (a) Prove that the random process $[X(t)]$ with constant mean is mean ergodic, if

$$\lim_{T\to\infty} \int_{-T}^{T}\int_{-T}^{T} \frac{C(t_1, t_2)}{4T^2}\, dt_1 dt_2 = 0.$$ (16)

Or

(b) (i) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t+\Theta)$ where Θ is uniformly distributed over $(-\pi,\pi)$. Prove that $\{X(t)\}$ is correlation ergodic. (8)

(ii) Find the mean and autocorrelation of the Poisson process. (8)

14. (a) State and Prove Wiener-Khintchine theorem, and hence find the power Spectral density of a WSS process *X(t)* which has an autocorrelation

$$R_{xx}(\tau) = A_0\left[1 - \frac{|\tau|}{T}\right], \quad -T \leq \tau \leq T. \qquad (16)$$

Or

(b) (i) If {X(t)} is a WSS process with auto correlation function $R_{XX}(\tau)$ and if Y(t) = X(t+a)-X(t-a), show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$. (8)

(ii) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}$. (8)

15. (a) (i) Prove that if the input to a time-invariant , stable linear system is a WSS process, then the output will also be a WSS process. (8)

(ii) If the input x(t) and the output y(t) are connected by the differential equation $T\frac{dy(t)}{dt} + y(t) = x(t)$, prove that they can be related by means of a convolution type integral, assuming that x(t) and y(t) are zero for $t \leq 0$. (8)

Or

(b) (i) Consider a system with transfer function $\frac{1}{1 + i\omega}$. An inpit signal with auto correction function $m\,\delta(\tau) + m^2$ is fed as input to the system. Find the mean and mean square value of the output. (8)

(ii) If $\{X(t)\}$is a WSS process and if $Y(t) = \int_{-\alpha}^{\alpha} h(u) \times (t-u)$ then prove that $R_{xy}(\tau) * h(\tau)$. (8)

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