| С                                     |   | Reg. No. :                             |                              |                         |                      |                       |     |       |                         |        |       |       |  |
|---------------------------------------|---|--|------------------------------|-------------------------|----------------------|-----------------------|-----|-------|-------------------------|--------|-------|-------|--|
|                                       | Question Paper Code: 51Z24  |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| M.E. DEGREE EXAMINATION, NOV 2018     |   |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| First Semester                        |   |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| CAD / CAM                             |   |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| 15PMA124 - ADVANCED NUMERICAL METHODS |   |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| (Regulation 2015)                     |   |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| Dur                                   | ation: Three hours  |  |                              |                         |                      |                       | Max | kimu  | m: 1                    | 00 N   | larks | 3     |  |
| Answer ALL Questions                  |   |  |                              |                         |                      |                       |     |       |                         |        |       |       |  |
| 1.                                    | PART - A (5 x $1=5$ Marks)<br>"As soon as a new value for a variable is formed by iteration, it is<br>used immediately in the following equations." |  |                              |                         |                      |                       |     |       |                         | )1- R  |       |       |  |
|                                       | (a) Gauss – Seidel method   |  |                              |                         | (b) Thomas algorithm |                       |     |       |                         |        |       |       |  |
|                                       | (c) Gauss – Jacobi met  | (0                                     | (d) Gauss elimination method |                         |                      |                       |     |       |                         |        |       |       |  |
| 2.                                    | The Error term in Adam – Bash forth Predictor formula is  |  |                              |                         |                      |                       |     |       |                         |        | CC    | )2 -R |  |
|                                       | $(a)\frac{14 h}{45}\Delta^4 y_0$  | $(b)\frac{14h}{45}\Delta^4 y_0$        | (0                           | $(\frac{14}{45})$       | $\frac{h}{\Delta^4}$ | <i>y</i> <sub>0</sub> |     | (d) ] | None                    | e of t | the a | bove  |  |
| 3.                                    | When explicit method is stable only if?   |  |                              |                         |                      |                       |     |       |                         |        | CC    | )3- R |  |
|                                       | (a) $\lambda > 1$   | (b) $\lambda > 1/2$                    | (0                           | c)λ<                    | < 1/2                |                       |     | (d) 2 | λ < 1                   |        |       |       |  |
| 4.                                    | The PDE $xf_{xx} + y f_{yy}$  | h = 0 is elliptic when                 |                              |                         |                      |                       |     |       |                         |        | CC    | )4 -R |  |
|                                       | (a) $x > 0$ and $y < 0$   |  |                              | (b) $x < 0$ and $y < 0$ |                      |                       |     |       |                         |        |       |       |  |
|                                       | (c) $x < 0$ and $y > 0$   |  |                              | (d) None of the above   |                      |                       |     |       |                         |        |       |       |  |
| 5.                                    | R(x) is orthogonal then   | 1                                      |                              |                         |                      |                       |     |       |                         |        | CC    | )5- R |  |
|                                       | (a) $\int_{0}^{1} R(x) F_{i}(x) dx = 0$   | (b) $\int_{-1}^{1} R(x) F_i(x) dx = 0$ | ) (0                         |                         | R(x)                 | dx =                  | 0   | (d)   | $\int_{0}^{1} F_{i}(z)$ | x)dx   | = 0   |       |  |

## $PART - B (5 \times 3 = 15 \text{ Marks})$

| 6.  | Write down formula for the Faddeev – Leverrier method?                         | CO1-U |
|-----|--|-------|
| 7.  | Write down the Runge-Kutta formula of fourth order.                            | CO2-U |
| 8.  | Give an example of parabolic equation.   | CO3-U |
| 9.  | Write down the five point finite difference scheme to solve Laplace equations. | CO4-U |
| 10. | Write formula for Galerkin Finite element method.                              | CO5-U |

$$PART - C (5 \times 16 = 80 Marks)$$

11. (a) (i) Evaluate  $\sqrt{12}$  to four decimal places by Newton's – Raphson CO1- App (8) method.

(ii) Solve by Gauss elimination method, the equations CO1- App (8) 2x + 3y = 7 = 5

$$2x + 3y - z = 5$$
  
 $4x + 4y - 3z = 3$   
 $-2x + 3y - z = 1$ 

Or

(b) (i) Solve the system of equations using pivot techniques CO1- App (8)

$$x+y+z = 7$$
  
 $3x+3y+4z = 24$   
 $2x+y+3z = 16$ 

(ii) Using Gauss-Seidel iterative method, solve the following CO1- App (8) system of equations:

8x-3y+2z = 30; 4x+11y-z = 33; 6x+3y+12z = 35.

12. (a) Find y (0.2) by Runge kutta method of fourth order if y'' - x y' = 0, CO2- App (16) y(0) = 1, y'(0) = 0

Or

(b) (i) Solve the equation y''(x) - xy(x) = 0 for  $y(x_i), x_i = 0, 1/3, 2/3$ , given that y(0) + y'(0) = 1 and y(1) = 1. (ii) Using Adam's Bash forth method find y(4.4) given  $5xy' + y^2 = 2, y(4) = 1, y(4.1) = 1.0049,$  y(4.2) = 1.0097 and y(4.3) = 1.0143. CO2- App (8) CO2- App (8)

- 13. (a) (i) Solve by Crank-Nicholson method,  $u_{t} = \frac{1}{16}u_{xx} \ 0 < x < 1, t > 0; u(x, 0) = 0, u(0, t) = 0, u(1,t) = 100t.$ Compute u for one time with h = 1/4. (ii) Explain implicit method Or (8)
  - (b) (i) Discuss the stability of two dimensional heat equation CO3-U (8)  $u_t = \alpha (u_{xx} + u_{yy}).$ (ii) Discuss ADI method to solve the two dimensional parabolic CO3-U (8) equations.
- 14. (a) Obtain a finite difference scheme to solve the Laplace equation. CO4 -App (16) Solve ∇<sup>2</sup>u = 0 at the pivotal points in the square shown fitted with square mesh. Use Leibmamm's iteration procedure. (5 iteration only)



Or

(b) Solve the Poisson's equation

CO4 - App (16)

 $u_{xx} + u_{yy} = -81xy$ , 0 < x < 1, 0 < y < 1 given that u(0, y) = 0, u(x, 0) = 0, u(1, y) = 100, u(y, 1) = 100 and h = 1/2

u(0,y) = 0, u(x,0) = 0, u(1,y) = 100, u(x,1) = 100 and h = 1/3.

## 15. (a) Solve the boundary value problem $u_{xx} + u_{yy} = -1, |x| \le 1, |y| \le 1$ and u=0 on |x| = 1, |y| = 1. Use the Galerkin finite element method to determine the solution values at the nodes $(0,0), (\frac{1}{2}, 0)$ and $(\frac{1}{2}, \frac{1}{2})$ . (Or)

(b) Solve the boundary value problem

CO5-App (16)

 $u_{xx} + u_{yy} = -2$ ,  $|x| \le 2$ ,  $|y| \le 2$  and u=0 on the boundary. Use the Galerkin finite element method to determine u at the nodes (0,0), (1,0) and (1,1).