Reg. No. : | | | | | | | | | | |

Question Paper Code: 44024

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2018

Fourth Semester

Electronics and Communication Engineering

14UMA424 - PROBABILITY AND RANDOM PROCESS

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. The probability of impossible event is
   (a) 1 (b) 0 (c) 2 (d) 0.5

2. The mean value of Poisson distribution is
   (a) θ (b) 1 (c) 0 (d) λ

3. If two random variables X and Y are independent, then covariance is
   (a) θ (b) 1 (c) 0 (d) λ

4. The conditional distribution of X given Y is
   (a) $f(x/y) = f(x, y) / f(x)$ (b) $f(x/y) = f(x, y) / f(y)$
   (c) $f(x/y) = f(x, y)\, f(x)$ (d) $f(x/y) = f(x, y)\, f(y)$

5. The sum of two independent Poisson process is
   (a) poisson process (b) marcov process
   (c) random process (d) stationary

6. If both T and S are discrete, then the random process is called
   (a) stationary (b) discrete random sequence
   (c) random process (d) poisson process

7. $R_{XX}(\tau)$ is an _________ function of τ
   (a) positive (b) 1 (c) even (d) odd

8. If $R_{xy}(\tau) = \mu_X \times \mu_Y$ then *X(t)* and *Y(t)* are called

   (a) Independent (b) Orthogonal (c) Stationary (d) none of these

9. Which of the following system is Causal?

   (a) $y(t)=x(t+a)$ (b) $y(t)= x(t-a)$

   (c) $(t)= a\, x(t+a)$ (d) $y(t)= x(t) - x(t-a)$

10. Colouted Noise means a noise that is

   (a) white (b) not white (c) coloured (d) none of these

PART - B (5 x 2 = 10 Marks)

11. If a Random Variable X has the Moment generating function $M_x(t)=\frac{2}{2-t}$. Determine the variance of X.

12. Let *X* and *Y* be random variables with joint density function $f(x,y)=2-x-y$ in $0 \le x < y \le 1$, fomulate the value of $E(x)$?

13. Outline discrete random process. Give an example for it.

14. State Winear–Khinchine theorem.

15. If $X(t)$ is a WSS process and if $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$, then formulate $R_{XY}(\tau) = R_{XY}(\tau) * h(\tau)$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) If the probability density function of a random variable *X* is given by $f(x) = K x^2 e^{-x}, x \ge 0$. Identify the value of *K*, r^{th} moment, mean and variance. (8)

   (ii) Establish the memory less property of geometric distribution. (8)

Or

   (b) A random variable *X* has the following probability function

Value of x	0	1	2	3	4
$P(x)$	k	$3k$	$5k$	$7k$	$9k$

   Find the value of k, $P(x < 3)$ and distribution function of x. (16)

17. (a) (i) The joint probability density function of a bivariate random variable *(X, Y)* is $f(x,y) = \begin{cases} k(x+y), 0<x<2, 0<y<2 \\ 0 \quad , elsewhere \end{cases}$. Find (1) the value of *k* (2) the marginal probability density of *x* and *y* (3) *x* and y independent. (8)

(ii) The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. The variance of *x* is 9. Evaluate the mean values of *x* and *y* and the Correlation coefficient between *x* and *y*. (8)

Or

(b) (i) The joint probability distribution of *X* and *Y* is given below:

Y \ X	-1	1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient between *X* and *Y*. (8)

(ii) If the pdf of a two dimensional random variable *(X, Y)* is given by $f(x, y) = x + y$, $0 < x, y < 1$. Find the pdf of $U = XY$. (8)

18. (a) (i) Enumerate that the process $X(t)=A\cos\omega t+B\sin\omega t$ is Wide –Sense Stationary ,where A and B are random variables if $E(A)=E(B)=0$ and $E(A^2)=E(B^2)$ and $E(AB)=0$ (8)

The transition probability matrix of a Markov chain $\{X_n\}$,n = 1,2,3,…. having three states1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$.

Identify i) $P[X_2=3]$, ii) $P[X_3=2,X_2=3,X_1=3,X_0=2]$ (8)

Or

(b) Generalize the Postulates of a Poisson process and Derive the Probability distribution for the Poisson Process. Also Show that the sum of two independent Poisson process is again a Poisson process. (16)

19. (a) (i) Define cross-correlation function and write the properties of cross-correlation function. (8)

(ii) State and Prove Wiener-Khinchine theorem. (8)

Or

(b) (i) Two random process *X(t)* and *Y(t)* are defined by $X(t) = A\cos\omega_0 t + B\sin\omega_0 t$ and $Y = B\cos\omega_0 t - A\sin\omega_0 t$. Show that *X(t)* and *Y(t)* are jointly wide-sense stationary, if *A* and *B* are uncorrelated random variables with zero means and the same variances and ω_0 is a constant. (8)

(ii) If the power spectral density of a WSS process is given by

$$S(\omega) = \begin{cases} \frac{b}{a}\left(a - |\omega|\right), & for\ |\omega| \le a \\ 0, & for\ |\omega| > a \end{cases}$$

Find the autocorrelation function of the process. (8)

20. (a) If *{X(t)}* is a WSS process and if $y(t) = \int_{-\infty}^{\infty} h(u) X(t-u)\,du$ then Prove that

(i) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$

(ii) $R_{YY}(\tau) = R_{XY}(\tau) * h(\tau)$

(iii) $S_{XY}(\omega) = S_{XX}(\omega) H^*(\omega)$

(iv) $S_{YY}(\omega) = S_{XX}(\omega) |H(\omega)|^2$ (16)

Or

(b) (i) The input to a time- invariant, stable linear system is a WSS process, Enumerate that the output will also be a WSS process. (8)

(ii) If *X(t)* is a band limited process such that $S_{xx}(\omega) = 0, |\omega| > \sigma$, then formulate $2[R_{xx}(0) - R_{xx}(\tau)] \le \sigma^2 \tau^2 R_{xx}(0)$. (8)
