Reg. No. : | | | | | | | | | | |

Question Paper Code: 51102

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

First Semester

Civil Engineering

15UMA102 – ENGINEERING MATHEMATICS - I

(Common to ALL Branches)

(Regulation 2015)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. $\lim_{x \to \infty} \frac{\log x}{\cot x}$

   (a) 2 (b) 0 (c) 2 (d) -2

2. $d(\tan^{-1} x) =$

   (a) $\frac{1}{x}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{1}{1+x}$ (d) $\frac{1}{1-x^2}$

3. $\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy =$

   (a) du (b) nu (c) udu (d) ∂u

4. If *u (x, y)* and *v (x, y)* are functionally independent then $\frac{\partial(u,v)}{\partial(x,y)} \neq$

   (a) 2 (b) 3 (c) 0 (d) -1

5. $\beta(m,n) =$

   (a) $\Gamma(m+n)$ (b) $\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

   (c) $\Gamma(m)\Gamma(n)$ (d) $\Gamma(m) + \Gamma(m)$

6. If $f(x)$ is odd then $\int_{-1}^{1} f(x)dx$.

(a) 2 (b) 0 (c) 1 (d) 1/2

7. Change the order of the integration of $I = \int_0^1 \int_0^x dydx$.

(a) $\int_0^1 \int_0^x dxdy$ (b) $\int_0^x \int_0^1 dydx$

(c) $\int_0^1 \int_y^1 dxdy$ (d) $\int_0^1 \int_0^x dydx$

8. Evaluate $\int_{-1}^{2} \int_{x}^{x+2} dydx$

(a) 2 (b) 0 (c) 6 (d) 1/2

9. If |A| = 0 then at least one of the Eigen values of *A* is

(a) positive (b) negative (c) 1 (d) 0

10. If *A* is an orthogonal matrix then

(a) |A| =0 (b) A is singular (c) $A^2 = I$ (d) $A' = A^{-1}$

PART - B (5 x 2 = 10 Marks)

11. Find $d(log\ [cos\ (log\ x)])$.

12. Find the stationary values of $f(x\ ,y) = x^3 + y^3 - 3axy$.

13. Prove $\frac{\beta(m+1,\ n)}{\beta(m,\ n+1)} = \frac{m}{n}$.

14. Evaluate $I = \int_0^{\pi} \int_0^{a\sin\theta} rdrd\theta$.

15. Two Eigen values of the matrix for A = $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ are 1 and 2. Find the third Eigen value and |A|.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the *nth* derivatives of $\frac{2x-3}{x^2-3x+2}$. (8)

(ii) If $y = 2\cos t - \cos 2t$, $x = 2\sin t - \sin 2t$. Find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{2}$. (8)

Or

(b) (i) A pizza, heated to a temperature of 400° Fahrenheit, is taken out of an oven and placed in a 75°F room at time t=0 minutes. The temperature of the pizza is changing such that its decay constant, k, is 0.325. At what time is the temperature of the pizza 150°F and, therefore, safe to eat? Give your answer in minutes. (8)

(ii) Find the first three non-zero terms of the Maclaurin series for $f(x) = e^x \cos x$. (8)

17. (a) (i) Find the greatest and the least distances of the point (3, 4, 12) from the unit sphere whose centre is at the origin. (8)

(ii) If $u = tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then prove $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$. (8)

Or

(b) (i) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ upto third degree terms. (8)

(ii) If $x + y + z = u$, $y + z = uv$, $z = uvw$, prove $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2v$. (8)

18. (a) (i) Evaluate $\int \frac{dx}{x^2-6x+13}$. (8)

(ii) Evaluate $I = \int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$. (8)

Or

(b) (i) Evaluate $I = \int_0^{\pi/2} \log\sin\theta\, d\theta$. (8)

(ii) Evaluate $I = \int_0^{\pi} \frac{x\sin^3 x}{1+\cos^2 x} dx$. (8)

19\. (a) (i) Change the order of integration $\int_0^1 \int_x^{2-x} \frac{x}{y} dy\, dx$ and hence evaluate. (8)

(ii) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by converting to polar co-ordinates. (8)

Or

(b) (i) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$. (8)

(ii) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{(x+y+z)} dz\, dy\, dx$. (8)

20\. (a) (i) Using Cayley - Hamilton theorem find A^{-1} *and* A^4 for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. (8)

(ii) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. (8)

Or

(b) Reduce the Q.F $10x^2 + 2y^2 + 5z^2 + 6yz - 10\,zx - 4xy$ into a Canonical form by an orthogonal transformation and hence find the rank, signature, index and nature of the Q.F. (16)
