Reg. No. : | | | | | | | | | | |

Question Paper Code: 41102

B.E. / B.Tech. DEGREE EXAMINATION, DECEMBER 2014.

First Semester

Civil Engineering

14UMA102 - ENGINEERING MATHEMATICS – I

(Common to all branches)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions.

PART A - (10 x 1 = 10 Marks)

1. If the Eigen values of the matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$ are 2, -2 then the Eigen values of A^T are

   (a) $\frac{1}{2}, \frac{-1}{2}$ (b) 2, -2 (c) 1, -1 (d) 1, 3

2. If two of the Eigen values of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ are 2 and 8, then the third Eigen value is

   (a) -2 (b) 0 (c) 2 (d) 3

3. Examine the nature of the series $1 + 2 + 3 + 4 + \ldots\ldots\ldots + n + \ldots\ldots\infty$

   (a) divergent (b) convergent (c) oscillatory (d) linear

4. The geometric series $1 + r + r^2 + r^3 + \ldots\ldots + r^n + \ldots\ldots$ converges if

   (a) $r \leq 1$ (b) $r \geq 1$ (c) $r > 1$ (d) $r < 1$

5. What is the radius of curvature at (3, 4) on the curve $x^2 + y^2 = 25$?

(a) 5 (b) -5 (c) 25 (d) -25

6. The envelope of the family of straight lines $y = mx + \frac{1}{m}$, m being the parameter is

(a) $y^2 = -4x$ (b) $x^2 = 4y$ (c) $y^2 = 4x$ (d) $x^2 = -4y$

7. If $u = x^3 y^2 + x^2 y^3$, where $x = at^2$ and $y = 2at$, then $\frac{du}{dt} =$

(a) $8t^6(4t + 7)$ (b) $8a^5 t^6(4t + 7)$ (c) $8t^6(4t + 7)$ (d) $a^5 t^6(4t + 7)$

8. If $u = \frac{y}{z} + \frac{z}{x}$ then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is

(a) u (b) –u (c) 2u (d) 0

9. $\int_0^1 \int_0^2 \int_0^3 xyz\,dx\,dy\,dz$

(a) 9 (b) $\frac{9}{4}$ (c) $\frac{9}{2}$ (d) $\frac{1}{9}$

10. By changing the order of integration, we get $\int_0^1 \int_0^y f(x, y)\,dx\,dy =$

(a) $\int_0^1 \int_0^x f(x, y)\,dy\,dx$ (b) $\int_0^1 \int_x^1 f(x, y)\,dy\,dx$ (c) $\int_0^1 \int_y^1 f(x, y)\,dx\,dy$ (d) $\int_0^1 \int_x^0 f(x, y)\,dy\,dx$

PART - B (5 x 2 = 10 Marks)

11. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.

12. Test the convergence of the series $\sum_1^{\infty} \frac{n!2^n}{n^n}$ by D'Alembert's Ratio test.

13. Find the envelope of the family of circles $(x - \alpha)^2 + y^2 = r^2$, where α being the parameter.

14. If $x = u^2 - v^2$ and $y = 2uv$, find the Jacobian of x and y with respect to u and v.

15. Evaluate $\int_0^2 \int_0^{\pi} r \sin^2\theta \, d\theta \, dr$.

PART - C (5 x 16 = 80 Marks)

16. (a) Diagonalize the matrix by orthogonal transformation $\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$. (16)

Or

(b) Reduce the Quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$ to canonical form through an orthogonal transformation and hence show that it is positive definite. (16)

17. (a) (i) Show that the sum of the series $\frac{15}{16} + \frac{15}{16} \times \frac{21}{24} + \frac{15}{16} \times \frac{21}{24} \times \frac{27}{32} + \cdots \infty = \frac{47}{9}$. (8)

(ii) Show that the series $1 - 2 + 3 - 4 + \ldots \infty$ oscillates infinitely. (8)

Or

(b) Show that the series $\sum_{h=1}^{\alpha} \frac{(-1)^{n-1}}{2n-1}$ is conditionally convergent. (16)

18. (a) (i) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. (8)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the evolute of the curve $x = a(\cos t + t \sin t)$, $y = a(\sin t - t\cos t)$. (8)

(ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. (8)

19. (a) (i) Find the Taylor's series of $e^x \log(1 + y)$ in powers of x and y up to third degree terms. (8)

(ii) Find the maximum value of $f(x, y) = \sin x \sin y \sin(x + y); 0 \le x, y < \pi$.. (8)

Or

(b) If u is function of x and y; by changing to polar form with $x = r\cos\theta$, $y = r\sin\theta$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2}$. (16)

20. (a) (i) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$. (8)

(ii) Evaluate $\iiint \frac{dx\ dy\ dz}{\sqrt{1 - x^2 - y^2 - z^2}}$ for all positive values of x, y, z for which the integral is real. (8)

Or

(b) (i) Find by double integration, the area between the two parabolas $y^2 = 9x$ and $x^2 = 9y$ (8)

(ii) By transforming into polar coordinates, evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dxdy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, $b > a$. (8)
