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Question Paper Code: 41404

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Fourth Semester

Electronics and Communication Engineering

14UMA424 - PROBABILITY AND RANDOM PROCESS

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. If X is a Random Variable, where Var (x) =4, then predict the Var (3X+8)

   (a) 36 (b) 0 (c) 26 (d) 44

2. The mean value of Poisson distribution is

   (a) θ (b) 1 (c) 0 (d) λ

3. When will the two Regression Lines be coincide

   (a) r=0 (b) r=1 (c) r= ± 1 (d) r= ∞

4. The conditional distribution of X given Y is

   (a) $f(x/y) = f(x, y) / f(x)$ (b) $f(x/y) = f(x, y) / f(y)$

   (c) $f(x/y) = f(x, y)\, f(x)$ (d) $f(x/y) = f(x, y)\, f(y)$

5. Every Strongly stationary process of order 2 is a

   (a) Orthogonal process (b) Stationary Process

   (c) WSS Process (d) None of these

6. If both T and S are discrete, then the random process is called

   (a) stationary (b) discrete random sequence

   (c) random process (d) Poisson process

7. Autocorrelation function is an ___________ function.

(a) odd (b) even
(c) neither Even nor odd (d) stationary

8. The power spectral density of $X(t)$ is defined by

(a) $y(t) = \int_{-\infty}^{\infty} h(u) X(t-u) du$ (b) $X(t) = \int_{-\infty}^{\infty} h(u) X(t-u) du$

(c) $s_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{i\omega\tau} d\tau$ (d) $s_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-i\omega\tau} d\tau$

9. Which of the following system is Causal?

(a) $y(t)=x(t+a)$ (b) $y(t)= x(t-a)$
(c) $(t)= a\, x(t+a)$ (d) $y(t)= x(t) - x(t-a)$

10. White noise is also called as
(a) system (b) white Gaussian noise
(c) functional white noise (d) stationary

PART - B (5 x 2 = 10 Marks)

11. If a Random variable X has the moment generating function $M_x(t)=\frac{2}{2-t}$. Determine the variance of X.

12. Define covariance.

13. Outline discrete random process. Give an example for it.

14. State Winear–Khinchine theorem.

15. If $X(t)$ is a WSS process and if $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$, then formulate $R_{XY}(\tau) = R_{XY}(\tau) * h(\tau)$.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) If the probability density function of a random variable X is given by $f(x) = K x^{2} e^{-x}$, $x \geq 0$. Identify the value of K, r^{th} moment, mean and variance. (8)

(ii) Establish the memory less property of geometric distribution. (8)

Or

(b) (i) Deduce the moment generating function of exponential distribution and hence find its mean and variance. (8)

(ii) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are taken at random from this set, interpret the probability that at least one of them would have scored above 75?(Given the area between z=0 and z=2 under the standard normal curve is 0.4772). (8)

17. (a) (i) If the joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$.

Find out (i) P(X > 1) , (ii) P(Y< ½). (8)

(ii) Analyse the correlation coefficient between the heights (in inches) of fathers *X* and their sons *Y* from the following data. (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

Or

(b) (i) The two lines of regression are $8x - 10y + 66 = 0$, $40x - 18y - 214 = 0$. The variance of x is 9. Evaluate the mean values of x and y and the Correlation coefficient between x and y. (8)

(ii) If *X* and *Y* are independent variants uniformly distributed in (0, 1). Identify the distribution of *XY*. (8)

18. (a) (i) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide sense-stationary, if A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 square parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (8)

Or

(b) (i) Explain the classification of random process. (8)

(ii) The transition probability of a Markov chain $\{X\}$, $n = 1, 2, 3$……., having 3 states 1, 2 and 3 $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (1) $P\{X_2 = 3\}$ and (2) $P = \{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$. (8)

19. (a) (i) Trace the power spectral of a random binary transmission process where auto correlation function $R_{xx}(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, for\ |\tau| \le T \\ 0, \quad for\ |\tau| > T \end{cases}$. (8)

(ii) If the power spectral density of a WSS process is given by $S_{xy}(\omega) = \begin{cases} a + jb\omega\ , |\omega| < 1 \\ 0, \quad elsewhere \end{cases}$. Evaluate the Cross correlation function. (8)

Or

(b) (i) State and Prove Wiener-Khinchine theorem. (8)

(ii) Given that the autocorrelation function of a stationary random process is $R_{xx}(\tau) = \frac{25\tau^2+36}{6.25\tau^2+4}$. Predict the mean and variance of the process {X(t)}. (8)

20. (a) (i) Show that $S_{yy}(\omega) = S_{xx}(\omega)|H(\omega)|^2$ where Sxx(ω) and Syy(ω) are the power spectral density functions of the input *X(t)*, output *Y(t)* and *H(ω)* is the system transfer function. (8)

(ii) If the input to a time- invariant, stable linear system is a WSS process, Enumerate that the output will also be a WSS process. (8)

Or

(b) (i) If *{X(t)}* is a Gaussian process with $\mu(t) = 10 \quad and \quad C(t_1, t_2) = 16\ e^{-|t_1 - t_2|}$. Find the probability of $X(10) \le 8$. (6)

(ii) If *Y (t) = A cos (ω₀t + θ) + N(t)* , where *A* is a constant , θ is a random variable with a uniform distribution in *(-π, π)* and *{N(t)}* is band limited Gaussian white noise with a power spectral density

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2}, & for\ |\omega - \omega_0| \le \omega_B \\ 0, & elsewhere \end{cases}$$

Find the power spectral density of *{Y(t)}*. Assume that *N(t)* and θ are independent. (10)