Reg. No. : | | | | | | | | | | |

Question Paper Code: 41044

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Fourth Semester

Electronics and Communication Engineering

01UMA424 - PROBABILITY AND RANDOM PROCESSES

(Regulation 2013)

Duration: Three hours Maximum: 100 Marks

(Statistical tables may be permitted)

Answer ALL Questions.

PART A - (10 x 2 = 20 Marks)

1. For a binomial distribution mean is 6 and S.D is $\sqrt{2}$. Find the first two terms of the distribution.

2. Define exponential distribution.

3. State the equations of the two regression lines. What is the angle between them?

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5. Define: Wide sense stationary random process.

6. Consider a Markov chain with state {0, 1} transition probability matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Is the state 0 periodic? If so, what is the period?

7. Define: Power spectrum.

8. Write down the Wiener-Khintchine theorem.

9. Define a system. When is it called as linear system?

10. Define White noise.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) A random variable X has p.d.f $f(x) = \begin{cases} kx^2 e^{-x}; x > 0 \\ 0, otherwise \end{cases}$. Find the r^th moment of X about origin. Hence find the mean and variance. (8)

(ii) A random variable X is uniformly distributed over (0, 10).
Find (i) $P(X < 3)$, $P(X > 7)$ and $P(2 < X < 5)$
(ii) $P(X = 7)$. (8)

Or

(b) (i) The density function of a random variable X is given by
$f(x) = Kx(2 - x), 0 \le x \le 2$. Find K, mean, variance and r^th moment. (8)

(ii) Define binomial distribution. Also obtain its moment generating function and hence the mean and variance. (8)

12. (a) If X and Y are the independent random variables with probability density function

$$f(x, y) = \begin{cases} 4xy\ e^{-(x^2 + y^2)}, & x \ge 0, y \ge 0 \\ 0 & elsewhere \end{cases}$$

Find the density function of $U = \sqrt{x^2 + y^2}$. (16)

Or

(b) (i) Calculate the correction coefficient for the following data. (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(ii) In a correction analysis, the equation of the two regression lines are $3x + 12y = 19$ and $3x + 9y = 46$. Find the value of the correction coefficient. (8)

13. (a) (i) If $\{X(t)\}$ is a WSS process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine the second order moment of the $RV\{X(8) - X(5)\}$. (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

Or

(b) (i) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t+\Theta)$ where Θ is uniformly distributed over $(-\pi,\pi)$. Prove that $\{X(t)\}$ is correlation ergodic. (8)

(ii) Find the mean and autocorrelation of the Poisson process. (8)

14. (a) State and Prove Wiener-Khintchine theorem, and hence find the power Spectral density of a WSS process $X(t)$ which has an autocorrelation

$$R_{xx}(\tau) = A_0\left[1 - \frac{|\tau|}{T}\right], \quad -T \le \tau \le T. \qquad (16)$$

Or

(b) (i) The autocorrelation function for a stationary process $X(t)$ is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean of random variable $Y = \int_0^2 X(t)\,dt$ and variance of $X(t)$. (8)

(ii) Consider two random process $X(t) = 3\cos(wt+\Phi)$ and $Y(t) = 2\cos(wt+\Phi-\pi/2)$ where Φ is a random variable uniformly distributed in $(0,2\pi)$. Prove that $\sqrt{R_{XX}(0)R_{YY}(0)} \ge |R_{XY}(\tau)|$. (8)

15. (a) (i) Show that if the input x(t) is a WSS process for a linear system, then output y(t) is a WSS process. (8)

(ii) If X(t) is the input voltage to a circuit and Y(t) is the output voltage. X(t) is a stationary random process with $\mu_x = 0$ and $R_{xx}(\tau) = e^{-2|\tau|}$. Find mean μ_y and power spectrum $S_{yy}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{i\omega + 2}$. (8)

Or

(b) (i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ and $S_{YY}(\omega)$ are the power spectral density functions of the input $X(t)$ and the output $Y(t)$ and $H(\omega)$ is the system transfer function. (8)

(ii) The input to the RC filter is a white noise process with ACF $R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$. If the frequency response $H(\omega) = \frac{1}{1 + j\omega RC}$, find the autocorrelation and the mean-square value of the output process Y(t). (8)