Reg. No. : | | | | | | | | | | |

Question Paper Code: 41401

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Fourth Semester

Computer Science and Engineering

14UMA421 - APPLIED STATISTICS AND QUEUEING NETWORKS

(Common to Information Technology)

(Regulation 2014)

Duration: Three hours Maximum: 100 Marks

Answer ALL Questions

(Statistical Tables are permitted)

PART A - (10 x 1 = 10 Marks)

1. The cdf of a random variable X is $F(x) = 1 - (1 + x)e^{-x}, x > 0$. The pdf of X is
   (a) x (b) xe^{x} (c) xe^{-x} (d) x^2

2. For the following density function $f(x) = ae^{-|x|}, -\infty < x < \infty$. The value of 'a' is
   (a) 1/2 (b) 0 (c) 1 (d) 2

3. If X and Y are independent , then f(x, y) =
   (a) f(x) (b) f(x)f(y) (c) f(y) (d) f(x) + f(y)

4. If $X_1, X_2, \ldots, X_n, \ldots$ is a sequence of independent RVs with $E(X_i) = \mu_i$ and $Var(X_i) = \sigma_i^2, i = 1,2, \ldots$ and if $S_n = X_1 + X_2 + \cdots + X_n$, then under certain general conditions S_n follows a
   (a) Binomial distribution (b) Poisson
   (c) Normal (d) Exponential

5. Expand R.B.D
   (a) Root Between Divisors (b) Real Numbers Between Divisors
   (c) Randomized Block Designed (d) Root Between Deviation

6. LSD is known to be suitable for a case when the number of treatments is between
   (a) 5 and 12 (b) 6 and 11 (c) 4 and 10 (d) 3 and 100

7. The traffic intensity in (M/M/1) : (∞ /FIFO) is

(a) $\rho = \lambda\mu$ (b) $\rho = \frac{\lambda}{\mu}$ (c) $\rho = \lambda^2/\mu$ (d) $\rho = \lambda + \mu$

8. Average time a customer waits before being served

(a) W_s (b) W_q (c) L_s (d) L_q

9. In Pollaczek – Khinchine formula, ρ =

(a) $\lambda E(T)$ (b) $\lambda^2 E(T)$ (c) $\lambda E^2(T)$ (d) $\lambda^2 E^2(T)$

10. No customer may enter the system from outside

(a) Jackson (b) Open Jackson

(c) Closed Jackson (d) None of these

PART - B (5 x 2 = 10 Marks)

11. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test in fewer than 4 trials?

12. The joint p.d.f of the RV (X, Y) is given by $f(x,y) = kxy\, e^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of k.

13. Write any two advantages of RBD?

14. Define Steady State and Transient state?

15. Define Open Jackson Networks?

PART - C (5 x 16 = 80 Marks)

16. (a) (i) A random variable *X* has the following probability distribution.

X	-2	-1	0	1	2	3
P(X)	0.1	*K*	0.2	2*K*	0.3	3*K*

(1) find the value of *K*, (2) Evaluate *P(X<2)* and *P(-2<X<2)*,

(3) obtain the mean of *X*. (8)

(ii) Derive MGF, Mean and Variance of Geometric Distribution. (8)

Or

(b) (i) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4 , find the probability that (i) a '1' is received and (ii) a '1' was transmitted given that a '1' was received. (8)

(ii) A random variable X has a pdf $f(x) = kx^2e^{-x}, x \geq 0$. Find k, mean and variance. (8)

17. (a) (i) Given $f_{xy}(x,y) = cx\,(x-y), 0 < x < 2, -x < y < x, and \quad 0$ elsewhere, (1) evaluate 'c' (2) find $f_x(x)$ (3) $f_{y/x}(y/x)$ and (4) $f_y(y)$. (8)

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of U = X – Y. (8)

Or

(b) (i) Let X and Y be random variables having joint density function $f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2), 0 \le x, y \le 1, \\ 0\,, elsewhere \end{cases}$. Find the correlation coefficient of (X,Y). (8)

(ii) A distribution with unknown mean μ has variance equal to 1.5. Use Central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. (P(|z| < 1.96) = 0.95). (8)

18. (a) The following data represent the number of units of production per day turned out by different workers using 4 different types of machines.

		Machine Type			
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(i) Test whether the five mean differ with respect to mean productivity

(ii) Test whether the mean productivity is the same for the four different machine types. ($F_{0.05}(4,12) = 3.26\,; F_{0.05}(3,12) = 3.49$). (16)

Or

(b) Analyze the variance in the Latin square of yields (in kgs) of paddy where P,Q, R, S denote the different methods of cultivation.

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether the different methods of cultivation have given significantly different yields. ($F_{0.05}\,(3,6) = 4.76$). (16)

19. (a) Arrivals of a telephone in a both are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assured to be distributed exponentially with mean 4 minutes.

(1) Find the average number of persons waiting in the system?

(2) What is the probability that a person arriving at the booth will have to wait in the queue?

(3) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

(4) Estimate the fraction of the day when the phone will be in use.

(5) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 minutes for phone. By how much the flow of arrivals should increase to justify a second booth?

(6) What is the average length of the queue that forms from time to time? (16)

Or

(b) A bank has 2 tellers working on savings accounts. The first teller handles withdrawals only . The second teller handles deposits only. It has been found that the service time distributions for both deposits and withdrawals are exponential with mean service time of 3 minutes per customer. Depositors are found to arrive rate of 16 per hour. Withdrawers also arrive in a Poisson fashion with arrival rate of 14 per hour.

(i) What would be the effect of the average waiting time for the customers if each teller could handle both withdrawals and deposits.

(ii) What would be the effect, if this could only be accomplished by increasing the service time to 3.5 minutes. (16)

20. (a) Derive Pollaczek – Khinchine Formula. (16)

Or

(b) In a library, there are 2 sections, one for English books and the other section for Tamil books. There is only one salesman in each section. Customers from outside arrive at the English book section at a Poisson rate of 5 per hour and at the Tamil book section at a Poisson rate of 4 per hour. The service rates of the English book section and Tamil book section are 9 and 11 per hour respectively. A customers after service at English book section is equally likely to go to the Tamil book section or to leave the library. However, a customer upon completion of service at Tamil book section will go the English book section with probability 1/3 and will leave the library otherwise. Find the

(i) Joint steady-state probability that there are 2 customers in the English book section and 2 in the Tamil book section.

(ii) Average number of customers in the library.

(iii) Average waiting time of a customer in the library. (16)
