Reg. No. : | | | | | | | | | | |

Question Paper Code: 51032

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Third Semester

Civil Engineering

15UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to EEE, ECE, EIE, Mechanical and Chemical Engineering Branches)

(Regulation 2015)

Duration: Three hours — Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. In the expansion of $x\cos x$ as a Fourier series in $(-l, l)$ the value of $a_n =$

   (a) 1 (b) -1 (c) l (d) 0

2. ________ cannot be expanded as a Fourier series.

   (a) $\sin x$ (b) $x - x^3$ (c) $\tan x$ (d) $\cos x$

3. If $F(s)= F[\ f(x)\ ]$, then $F[\ f(x-a)\ ]=$

   (a) $e^{isa} F(s)$ (b) $e^{-isa} F(s)$ (c) $e^{isx} F(s)$ (d) $e^{-isx} F(s)$

4. If $f(x) = xe^{-x^2/2}$ is self – reciprocal with respect to Fourier sine transform, then $F_s\left[xe^{-x^2/2}\right] =$

   (a) $xe^{-s^2/2}$ (b) $se^{-s^2/2}$ (c) $se^{-x^2/2}$ (d) $xe^{-sx^2/2}$

5. Z (1)=

   (a) $\frac{1}{z-1}$ (b) $\frac{1}{z}$ (c) $\frac{z}{z-1}$ (d) $\frac{z^2}{z-1}$

6. $\sum_{r=0}^{n} f(r)g(n-r) =$

(a) $f(n) * g(n)$ (b) $f(n)/g(n)$
(c) $f(n) \cdot g(n)$ (d) $f(n) - g(n)$

7. The partial differential equation obtained from $z = (x^2 + a^2)(y^2 + b^2)$ is

(a) $4yz = pq$ (b) $4xz = p + q$
(c) $4xyz = p - q$ (d) $4xyz = pq$

8. A solution that contains as many arbitrary constants as there are independent variables is called as

(a) singular integral (b) general integral
(c) complete integral (d) particular integral

9. If $B^2 - 4AC < 0$, then the second order partial differential equation is said to be

(a) parabolic (b) elliptic (c) hyperbolic (d) quadratic

10. In ______ state, temperature do not depend on time 't'.

(a) steady (b) transient (c) absolute (d) bounded

PART - B (5 x 2 = 10 Marks)

11. State Dirichlet's conditions for the existence of Fourier series.

12. Find the Fourier sine transform of $\frac{1}{x}, 0 < x < \infty$.

13. State initial and final value theorem on Z – transform.

14. Form the partial differential equation by eliminating the arbitrary function from $z = f(my - lx)$.

15. Write the three possible solutions of the one dimensional wave equation.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the Fourier series expansion of $f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$ and hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \cdots = \frac{\pi^2}{8}$. (8)

(ii) Express $f(x) = x(\pi - x), 0 < x < \pi$, as a Fourier sine series of periodicity 2π and deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots = \frac{\pi^3}{32}$. (8)

Or

(b) (i) Find the Fourier series of $y = x^2$ in $-\pi < x < \pi$ and show that

$1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$. (8)

(ii) The following table gives the variations of a periodic function over the period *T*.

x	*0*	*T/6*	*T/3*	*T/2*	*2T/3*	*5T/6*	*T*
y	*1.98*	*1.3*	*1.05*	*1.3*	*-0.88*	*-0.25*	*1.98*

Show that $f(x) = 0.75 + 0.37cos\theta + 1.004sin\theta$ where $\theta = \frac{2\pi x}{T}$. (8)

17. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence prove that

(i) $\int_0^\infty \left(\frac{sins - s\cos s}{s^3}\right) cos\frac{s}{2} ds = \frac{3\pi}{16}$. (8)

(ii) $\int_0^\infty \left(\frac{sinx - xcosx}{x^3}\right)^2 dx = \frac{\pi}{15}$. (8)

Or

(b) (i) Find the Fourier cosine transform of $\frac{1}{a^2+x^2}$. (8)

(ii) Find the Fourier Sine transform of xe^{-ax}. (8)

18. (a) (i) Find $Z\left(\frac{1}{n(n-1)}\right)$ (8)

(ii) Find $Z(r^n cosn\theta)$ and $Z(r^n sinn\theta)$. (8)

Or

(b) (i) Find $Z^{-1}\left(\frac{z^2}{(z-a)^2}\right)$ using convolution theorem. (8)

(ii) Solve the difference equation $Y_{n+2} - 5Y_{n+1} + 6Y_n = 4^n$ given that $Y_0=D$, $Y_1=1$.

(8)

19. (a) (i) Solve: $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. (8)

(ii) Solve: $(D^3 + D^2D' - DD'^2 - D'^3)z = e^{2x+y} + \cos(x+y)$. (8)

Or

(b) (i) Find the singular integral of $z = px + qy + p^2 - q^2$. (8)

(ii) Find the partial differential equation of all spheres whose radii are the same. (8)

20. (a) A tightly stretched string with fixed end points $x = 0$ *and* $x = l$ is initially in a position given by $y(x, 0) = k(lx - x^2), 0 < x < l$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t. (16)

Or

(b) An infinitely long plate of width π *cms* with insulated surfaces has its temperature $u = 0$ on both long sides and one of the shorter sides. The temperature along the short edge $y = 0$ is given by $u(x, 0) = 3x, 0 < x < \pi$. Find the steady state temperature distribution $u(x, y)$. (16)
