Reg. No. : | | | | | | | | | | |

Question Paper Code: 41031

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: Three hours   Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 1 = 10 Marks)

1. If *f(x)* is even in then the Fourier Coefficient bn =

   (a) 1   (b) 0   (c) π (d) $-\pi$

2. R.M.S value of f(x) = x in (-1,1) is

   (a) 0   (b) 1   (c) $\frac{1}{3}$ (d) $\sqrt{\frac{1}{3}}$

3. If $F(f(x)) = F(s)$ and $F(g(x)) = G(s)$ then $F(f(x) * g(x)) =$ _______

   (a) $F(s)G(s)$ (b) $F(s) * G(s)$ (c) $f(s) * g(s)$ (d) $F(x) * G(x)$

4. If $F(f(x)) = F(s)$ then $F(f(x-a)) =$ _______

   (a) $e^{aix}F(s)$ (b) $e^{-ais}F(s)$ (c) $e^{ais}F(s)$ (d) $e^{-aix}F(s)$

5. $\lim_{z \to 1}$ (z-1) F(z) =

   (a) f(1)   (b) $F(\infty)$ (c) $f(\infty)$ (d) f(0)

6. Z(1) =______

(a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{1}{z+1}$ (d) $\frac{1}{z-1}$

7. The two-dimensional heat equation is

(a) $\nabla^2 u = 0$ (b) $\nabla^2 u = u_{tt}$

(c) $u_{xx} + u_{yy} = 0$ (d) $u_t = a^2 (u_{xx} + u_{yy})$

8. Classify the partial differential equation $4u_{xx} = u_t$.

(a) Hyperbolic (b) parabolic (c) Elliptic (d) Poisson

9. The finite difference approximation to $y_i' =$

(a) $\frac{y_{i+1} - y_{i-1}}{h}$ (b) $\frac{y_{i+1} + y_{i-1}}{h}$

(c) $\frac{y_{i+1} - y_{i-1}}{2h}$ (d) $\frac{y_{i+1} + y_{i-1}}{2h}$

10. The Laplace equation is

(a) $\nabla^2 u = \nabla x$ (b) $\nabla^2 u = f(x, y)$

(c) $\nabla^2 u - f(x, y) = 0$ (d) $\nabla^2 u = 0$

PART - B (5 x 2 = 10 Marks)

11. Find the Fourier constant a_0 for $f(x) = k$ in $(0, 2\pi)$

12. State the Fourier integral theorem.

13. Find the Z – transform of f(n)= n.

14. Write all possible solutions of the two-dimensional heat equation in steady state.

15. Write down the standard five-point formula to solve the Laplace equation.

PART - C (5 x 16 = 80 Marks)

16. (a) (i) Find the Fourier series for $f(x) = 1 + x + x^2$ in $(-\pi, \pi)$. Deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \infty = \frac{\pi^2}{6}$. (12)

(ii) Find the half range sine series for f(x) = x in $(0, \pi)$. (4)

Or

(b) (i) Find the cosine series for $f(x) = \pi x - x^2$ in $(0, \pi)$ and show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots = \frac{\pi^4}{90}$. (8)

(ii) Find Fourier series up to second harmonic for the following data:

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1

(8)

17. (a) (i) Evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2 + a^2)(x^2 + b^2)}$ (8)

(ii) Find Fourier cosine transform of $e^{-a^2x^2}$ (8)

Or

(b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ .Hence deduce the value of

(i) $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2}\,ds$ (ii) $\int_0^\infty \left[\frac{\sin s - s\cos s}{s^3}\right]^2 ds$ (16)

18. (a) (i) Find the inverse Z – transform of $\frac{10z}{(z-1)(z-2)}$ (8)

(ii) State and prove convolution theorem on Z – Transform. (8)

Or

(b) (i) Solve $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$ *given that* $y_0 = 0$ *and* $y_1 = 0$ using Z- transform. (10)

(ii) Find Z- transform of $\frac{1}{(n+1)(n+2)}$. (6)

19. (a) If a string of length l is initially at rest in its equilibrium position and each of its Point is given a velocity $v_0 \sin^3\left(\pi x / l\right)$, $0 < x < l$, find the displacement of the string $y(x,t)$. (16)

Or

(b) A metal bar 20cm long, with insulated sides, has its ends A and B kept at $30°C$ and $90°C$ respectively until steady state conditions prevail. The temperature at each end is suddenly raised to $0°C$ and kept so. Find the subsequent temperature at any time of the bar at any time. (16)

20. (a) Solve the Poisson's equations $\nabla^2 u = -81xy$, $0 < x < 1$, $0 < y < 1$, h=1/3, *u(0,y)= u(x,0), u(1,y)=u(x,1)*=100. (16)

Or

(b) (i) Solve u_{xx}=32 u_t with h=0.25 for t>0; 0<x<1 and u(x,0)=u(0,t)=0; u(1,t)=t .Tabulate u upto t=5 sec using Bender-Schmidt formula. (8)

(ii) Find the solution to the wave equations $u_{xx} = u_{tt}$,0 < x < 1, t > 0, given that $u_t(x,0)=0$, *u(1,t) = u (0,t) = 0* and *u (x,0) = 100 sin* π *x.* Compute u for 4 time steps with *h*=0.25. (8)