Reg. No. : | | | | | | | | | | |

Question Paper Code: 31301

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Third Semester

Civil Engineering

01UMA321 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL Branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. State Parseval's theorem in Fourier series.

2. If the Fourier series of the function $f(x) = x + x^2$ in the interval $-\pi \le x \le \pi$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

3. Prove that if $F(s)$ is the Fourier transform of $f(x)$, then $F\{ f(x-a) \} = e^{isa} F(s)$.

4. Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$

5. Find Z transform of a^n.

6. Write the formula for $Z^{-1}[F(z)]$ using Cauchy's residue theorem.

7. Write down the three possible solutions of one dimensional heat equation.

8. Classify the PDE $f_{xx} - 2f_{xy} = 0$.

9. Write the diagonal five point formula to solve the equation $u_{xx} + u_{yy} = 0$.

10. State Crank – Nicholson scheme to solve $u_{xx} = a u_t$ when $k = ah^2$.

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Find the Fourier series of $f(x) = \begin{cases} 1 & in \quad (0, \pi) \\ 2 & in \quad (\pi, 2\pi) \end{cases}$ Hence find the sum of the series $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} ..... \infty$ (8)

(ii) Obtain the Fourier series of the function $f(x) = \begin{cases} 1 + x & in \quad 0 < x < \pi \\ x - 1 & in \quad -\pi < x < 0 \end{cases}$. (8)

Or

(b) (i) Find the Half range cosine series for $y = x$ in $(0, l)$ and hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ... \infty$. (8)

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ given by (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	0.8	0.6	0.4	0.7	0.9	1.1	0.8

12. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| : |x| < 1 \\ 0 : otherwise \end{cases}$ and hence find the value of $\int_0^\infty \frac{sin^4 t}{t^4}\, dt$ (8)

(ii) Find the Fourier cosine transform of e^{-x^2} and hence find the Fourier sine transform of $x\, e^{-x^2}$. (8)

Or

(b) (i) Find the Fourier transform of $e^{-a|x|}$ $if\ a > 0$ (8)

(ii) Find the Fourier sine transform of $f(x) = \begin{cases} x & : 0 < x < 1 \\ 2 - x & : 1 < x < 2 \\ 0 & : x > 2 \end{cases}$ (8)

13. (a) (i) Find $Z\left[\frac{2n+3}{n(n+1)}\right]$. (8)

(ii) Find $z^{-1}\left[\frac{Z}{Z^2+11Z+24}\right]$. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ using convolution theorem. (8)

(ii) Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0$, $y_0 = -1, y_1 = 2$. (8)

14. (a) A tightly stretched string of length l has its ends fastened at x = 0, x = l. The mid-point of the string is then taken to height h and then released from rest in that position. Find the lateral displacement of a point of the string at time t from the instant of release. (16)

Or

(b) The ends A and B of a rod l cm long have the temperature at 30^0c and 80^0c until steady state prevails. The temperature of the ends is then changed to 40^0c and 60^0c respectively. Find the temperature distribution in the rod at any time. (16)

15. (a) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $u = 0$ on the boundary and mesh length 1 unit. (16)

Or

(b) Solve numerically $4u_{xx} = u_{tt}$ with the boundary conditions $u(0,t) = 0$, $u(4,t) = 0$ and the initial conditions $u_t(x,0) = 0$ and $u(x,0) = x(4 - x)$ taking $h = 1$ up to *4* time steps. (16)