Reg. No. : | | | | | | | | | | |

Question Paper Code: 31202

B.E. / B.Tech. DEGREE EXAMINATION, NOV 2016

Second Semester

Civil Engineering

01UMA202 - ENGINEERING MATHEMATICS - II

(Common to ALL branches)

(Regulation 2013)

Duration: Three hours | Maximum: 100 Marks

Answer ALL Questions

PART A - (10 x 2 = 20 Marks)

1. Find the particular integral of $(D^2 + 4)y = \pi$.
2. Transform $[(2x+3)^2 D^2 - 2(2x+3)D - 12]\, y = 0$ into an ordinary differential equation.
3. State Green's theorem.
4. Find 'a' such that $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal.
5. Test the analyticity of the function $f(z) = \bar{z}$.
6. Prove that an analytic function with constant real part is constant.
7. Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ at $z = 2$.
8. Expand $\frac{1}{z-2}$ at $z = 1$ in a Taylor's series.
9. State and prove the shifting property in Laplace Transform.

10. If $L\,[f(t)] = \frac{s}{(s+2)^3}$, find the value of $\lim_{t \to 0} f(t)$

PART - B (5 x 16 = 80 Marks)

11. (a) (i) Solve: $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$. (8)

(ii) Solve $(D^2+4)\, y = x \sin x$. (8)

Or

(b) (i) Solve $(D^2+2D+5)\, y = e^{-x} \tan x$ by method of variation of parameter. (8)

(ii) Solve $\frac{dx}{dt} + y = \sin t$ and $\frac{dy}{dt} + x = \cos t$ given x = 2, y = 0 when t = 0. (8)

12. (a) (i) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. (8)

(ii) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F} = \Delta\phi$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = xz\vec{i} + 4xy\vec{j} - z^2\vec{k}$ over the cube bounded by $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$. (16)

13. (a) (i) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and determine its harmonic conjugate. (8)

(ii) Prove that the analytic function with constant modulus is also constant. (8)

Or

(b) (i) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into $w = i, 1, 0$. (8)

14. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z+1)(z+2)} dz$ where C is the circle $|z| = 3$. (8)

(ii) Using Residue theorem evaluate $\int_C \frac{z}{(z-1)(z-2)} dz$ where C is the circle $|z - 2| = 1/2$. (8)

Or

(b) (i) Obtain Laurent's series expansion for $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| < 2$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$, using Contour integration in the complex plane. (8)

15. (a) (i) Find the Laplace transform of a periodic function $f(t) = \begin{cases} t & 0 < t < 1 \\ 2 - t & 1 < t < 2 \end{cases}$ and $f(t) = f(t+2)$. (8)

(ii) Using Laplace transform, solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3t}$, given $y = 2$ and $\frac{dy}{dx} = 3$ when $t = 0$. (8)

Or

(b) (i) Use convolution theorem to find inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ (8)

(ii) Solve using Laplace transform $\frac{d^2y}{dx^2} + 9y = 18t$ given that $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$. (8)
