Reg. No. : | | | | | | | | | | |

Question Paper Code: 43021

B.E. / B.Tech. DEGREE EXAMINATION, DEC 2020

Third Semester

Civil Engineering

14UMA321 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to ALL branches)

(Regulation 2014)

Duration: One hour Maximum: 30 Marks

PART A - (6 x 1 = 6 Marks)

(Answer any six of the following questions)

1. The formula for finding the Euler constant a_n of a Fourier series in [0,2 π] is ___

   (a) $a_n = \int_0^{\pi} f(x)cosnx\, dx$ (b) $a_n = \int_0^{2\pi} f(x)cosnx\, dx$

   (c) $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)cosnx\, dx$ (d) $a_n = \frac{1}{\pi}\int_0^{\pi} f(x)cos\frac{n\pi x}{l}\, dx$

2. The complex form of Fourier series of *f(x)* in $(-\ell,\ell)$ is given by, *f(x)* = $\sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{l}}$, where C_n is,

   (a) $\frac{2}{l}\int_{-l}^{l} f(x)e^{-\frac{in\pi x}{l}}dx$ (b) $\frac{1}{2l}\int_{-l}^{l} f(x)e^{-\frac{in\pi x}{l}}dx$

   (c) $\frac{2}{l}\int_{0}^{l} f(x)e^{-\frac{in\pi x}{l}}dx$ (d) $\frac{1}{2l}\int_{0}^{l} f(x)e^{\frac{in\pi x}{l}}dx$

3. $F\left(e^{-|x|}\right)$

   (a) does not exist (b) $\frac{2is}{(s^2+1)}$ (c) $\frac{1}{\sqrt{2\pi}}\frac{s}{(s^2+1)}$ (d) $\frac{1}{\sqrt{2\pi}}\frac{2}{(s^2+1)}$

4. Fourier sine transform of *xf(x)* is,

   (a) $F_c'(s)$ (b) $F_s'(s)$ (c) $-F_c'(s)$ (d) $-F_s'(s)$

5. $Z(n(n-1))$

(a) $\frac{2z}{(z+1)^3}$ (b) $\frac{z(z-1)}{(z-1)^3}$ (c) $\frac{2z}{(z-1)^3}$ (d) $\frac{z}{(z-1)^2}$

6. $Z(\cos^2 t)$

(a) $\frac{z}{2(z-1)}+\frac{z(z-\cos 2T)}{2(z^2-2z\cos 2T+1)}$ (b) $\frac{z}{(z-1)}+\frac{z(z-\cos 2T)}{(z^2-2z\cos 2T+1)}$

(c) $\frac{z}{2(z-1)}-\frac{z(z-\cos 2T)}{2(z^2-2z\cos 2T+1)}$ (d) $\frac{z}{(z-1)}-\frac{(z-\cos 2T)}{(z^2-2z\cos 2T+1)}$

7. When the ends of a rod is non zero for one dimensional heat flow equation, the temperature function *u(x,t)* is modified as the sum of steady state and transient state temperatures. The transient part of the solution which,

(a) increases with increase of time (b) decreases with increase of time

(c) increases with decrease of time (d) increases with decrease of time

8. The two dimensional heat flow equation in steady state is,

(a) $\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$

(c) $\frac{\partial u}{\partial t}=\alpha^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ (d) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

9. In the explicit formula for solving one dimensional heat equation, λ is ____

(a) $\frac{a}{h^2}$ (b) ah (c) $\frac{k}{ah^2}$ (d) ka

10. The standard five point formula in solving Laplace equation over a region is,

(a) $u_{ij}=\frac{1}{4}\left(u_{i-1,j+1}+u_{i-1,j}+u_{i+1,j}+u_{i+1,j-1}\right)$ (b) $u_{ij}=\frac{1}{4}\left(u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}\right)$

(c) $u_{ij}=\frac{1}{4}\left(u_{i-1,j-1}+u_{i+1,j+1}+u_{i+1,j-1}+u_{i-1,j+1}\right)$ (d) $u_{ij}=\frac{1}{4}\left(u_{i-1,j}+u_{i-1,j-1}+u_{i+1,j}+u_{i+1,j-1}\right)$

PART – B (3 x 8= 24 Marks)

(Answer any three of the following questions)

11. Find the Fourier series for $f(x)=1+x+x^2$ in $(-\pi,\pi)$. Deduce that

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\infty=\frac{\pi^2}{6}$$ (8)

12. Find Fourier transform of $e^{-a^2x^2}$, $a>0$ and hence show that $e^{-x^2/2}$ is self-reciprocal. (8)

13. Find $Z\ (t^2 e^{-t})$ and $Z\ (\sin^3 \frac{n\pi}{6})$ (8)

14. A string is stretched between two fixed points at a distance 2ℓ apart and the points of the string are given initial velocities v where,

$$V = \begin{cases} \frac{cx}{l} \ in\ 0 < x < l \\ \frac{c(2l-x)}{l} \ in\ \ l < x < 2l \end{cases}$$

, x being the distance from an end point. Find the displacement of the string at any subsequent time. (8)

15. Solve $U_{xx} + U_{yy} = 0$, over the square mesh of side 4 units satisfying the following boundary conditions, by using Liebmann's iteration method by taking $h = k = 1$

(i) $U(0,y) = y^2/4$ for $0 \le y \le 4$

(ii) $U(4,y) = y^2$ for $0 \le y \le 4$

(iii) $U(x,0) = 0$ for $0 \le x \le 4$

(iv) $U(x,4) = 8 + 2x$ for $0 \le x \le 4$ (8)